Time-to-Build, Regulation, and Investment*

Haejun Jeon†
Tokyo University of Science

Abstract

This study investigates the effects of uncertainty in time-to-build and regulation, which hinders immediate revenue generation after investment, on a firm's optimal investment decision. We show that in the absence of regulation, uncertainty in time-to-build always accelerates investment and enhances firm value. We also show that in the absence of time-to-build, uncertainty in regulation can mitigate the distortion of investment induced by regulation. Furthermore, in the presence of both time-to-build and regulation, there can exist harmless regulation that does not induce any distortion in the investment decision and does not harm firm value. Lastly, in the presence of both time-to-build and regulation, not only uncertainty in time-to-build but also its presence can accelerate investment.

JEL Classification Codes: D25, G31, G38
Keywords: time-to-build, investment lags, regulation, deregulation, real options

*This version: May 24, 2024.

†Corresponding Author. Department of Management, Tokyo University of Science, 1-11-2 Fujimi, Chiyoda-ku, Tokyo, 102-0071, Japan; E-mail: jeon@rs.tus.ac.jp; Phone: 81-3-3288-2544, Fax: 81-3-3288-2498. This paper was written when the author was a visiting academic at The University of Queensland and he would like to express gratitude to Kazutoshi Yamazaki for kind hospitality. This study was supported by the JSPS KAKENHI (grant number 22K13434) and Tokyo University of Science Research Grants.

1 Introduction

Recently technological innovation has advanced at an unprecedented pace, and investment projects based on state-of-the-art technologies are exposed to increasing uncertainty. In particular, *the uncertainty of the timing of revenue generation* has increased notably, in addition to that of market demands that traditional studies have focused on. This can arise from the project's time-to-build, which is an internal factor. For instance, revenue generation from a chipmaker's investment in semiconductor fabrication plants can be delayed significantly due to the shortage of advanced lithography machines, which are necessary for manufacturing high-end chips (Hollinger and Waters (2022)).[1] The sluggish capacity expansion of the COVID-19 vaccines in the early stages of the pandemic is another recent example of uncertainty in the timing of revenue generation caused by production lags.[2] Furthermore, revenue generation can be delayed due to external factors such as regulation. The timing of commercialization can be highly uncertain in an unexplored field where relevant laws have not yet been established. For instance, robotaxis with fully autonomous driving have been tested in a few cities but mass market commercialization is still far off, mainly due to the lack of relevant legislation.[3] Drone delivery[4] and airtaxi[5] are additional examples of regulation holding back the development of new markets despite the technology almost in place.[6]

Some studies have considered uncertain time-to-build (e.g., Weeds (2002), Alvarez and Keppo (2002), Jeon (2021*a*,*b*, 2023)) in the discussion of a firm's optimal investment decision. However, none of them have clarified *the sheer effects of uncertainty* in time-to-build on corporate investment by comparing it with the case of constant time-to-build. The effects of regulation on investment have mainly been discussed by empirical analysis, most of which have found negative (e.g., Alesina et al. (2005), Klapper et al. (2006), Barone and Cingano (2011), Fabrizio (2013), Branstetter et al. (2014), Gulen and Ion (2016)). To the best of our knowledge, there is no theoretical study that clarifies the sheer effects of regulatory uncertainty on investment by comparing it with the case of certain regulation. In this study, we assume that an investment project does not yield revenue immediately after the investment due to time-to-build and/or regulation and investigate the effects of uncertainty in the timing of revenue generation on corporate investment based on the real options framework. First, we discuss the impacts of uncertainty in time-to-build in the absence of regulation by comparing the firm's optimal investment in a project with known and certain time-to-build to that with uncertain time-to-build. Next, we investigate the impacts of uncertainty in regulation in the absence of time-to-build by comparing the firm's optimal investment under a fixed-duration regulation to that under uncertain-duration regulation. Lastly, we analyze the effects of uncertainty in the timing of revenue generation in the presence of both time-to-build and regulation.

First, we show that *uncertainty in time-to-build always accelerates investment and enhances firm value* in the absence of regulation. That is, the investment threshold and firm value with uncertain time-to-build are

[1]The investment lags of semiconductors led to production lags in other industries as well. For instance, it led to the drop in production by Volkswagen and Stellantis, the two largest European car makers, with production decreasing by 35% and 30%, respectively, in the three months to September 2021 (Boston and Kostov (2021)).

[2]Global COVID-19 Vaccine Supply Chain and Manufacturing Summit noted that 837 million doses of vaccines were projected to be manufactured in 2020 but less than 4% of the original projection was made in 2020.

[3]On August 10 2023, the California Public Utilities Commission decided to allow Cruise and Waymo, mainly owned by General Motors and Google's parent Alphabet, respectively, to operate full commercial robotaxi services in San Francisco. This decision, however, has no bearing on other states in the U.S., and the lack of cohesive regulation is expected to hold back the robotaxi market for years to come (Financial Times Lex (2023)). Rather, the operation of one of them, Cruise, had been suspended since October 24 2023, only two months after the launch, due to many collisions with pedestrians.

[4]As of 2023, five drone delivery operators acquired air carrier certification called Part 135 from the U.S. Federal Aviation Administration (FAA): Wing Aviation, UPS Flight Forward, Amazon, Zipline, and Causey Aviation Unmanned. However, their operations have been greatly limited by stringent regulations, including the limitation of operation beyond visual line of sight (BVLOS), which implies that a human needs to see and steer away from other aircraft to prevent a possible crash. In September 2023, UPS Flight Forward and Zipline received authorization for BVLOS drone delivery in a specific area. However, nationwide delivery by autonomous drones is still far off.

[5]The chief executive of Archer Aviation, a leading company in electric vertical take-off and landing (eVTOL) aircraft industry, noted that European Union Aviation Safety Agency (EASA) regulation makes it extremely hard to bring new vehicles to the market; the U.S. FAA has not published any standard yet (Bushey and Pfeifer (2023)).

[6]Regulation hindering the market development of new technology is not a novel or recent phenomenon. The Locomotive Acts 1865 in the U.K., better known as Red Flag Acts, required that every automobile should be accompanied by three persons, one of which is obligated to precede the automobile on foot, carrying a red flag constantly to warn the riders and drivers of horses of the approach of the automobile. It was intended to protect the horse-drawn carriage industry, the existence of which was threatened by the rise of steam-powered vehicles. However, it is generally said that this notorious regulation stifled the growth of the U.K. auto industry in the 19th century.

always lower and higher than those with fixed time-to-build, respectively, provided that the size of time-to-build is identical in terms of expectation. This counterintuitive result comes from the fact that the discount factor is strictly convex with respect to the timing at which cash flows are generated. Thus, good news (i.e., cash flows generated earlier than expected) is discounted less and bad news (i.e., cash flows generated later than expected) is discounted more. This novel result is in sharp contrast with extant studies that focus on the effects of uncertainty in market demand (e.g., Hartman (1972, 1973), Abel (1983), McDonald and Siegel (1986), Pindyck (1988, 1993)), which are positive or negative depending on whether the marginal revenue product of capital is convex or not.[7] By contrast, this study focuses on a novel aspect of uncertainty in the market and reveals its strictly positive impact on investment and firm value.

We also show that *uncertainty in regulation can mitigate the distortion of investment induced by regulation* in the absence of time-to-build. That is, the state price of investment and firm value under uncertain-duration regulation can be higher than those under fixed-duration regulation, provided that the durations are identical in terms of expectation. Naturally, a firm would never invest in a project under regulation that hinders the market penetration of the project's result, whether the duration of regulation is uncertain or not. When the duration is significantly long, it is highly probable that the demand will grow enough before deregulation such that the firm would have invested if it had not been for the regulation, resulting in a distorted investment decision. In such cases, uncertainty in regulation can reduce the probability of investment delayed by regulation, which improves firm value.

Furthermore, we show that in the presence of both time-to-build and regulation, there can exist *harmless regulation that does not induce any distortion in the investment decision and does not harm firm value.* As mentioned above, investment under regulation in the absence of time-to-build is always suboptimal. However, it can be optimal for the firm to invest in a project that involves time-to-build under regulation, expecting that the project would be completed by the time the regulation is lifted. When the duration of both time-to-build and regulation is known with certainty, regulation lasting less than the length of time-to-build does not affect the firm's investment decision, and thus, does not harm firm value. In the real world, a strict regulation that prohibits the commercialization of radical technologies or products can be necessary to gain time for legal arrangements, even at the cost of delaying the introduction of new technologies to the market. This study shows the existence of harmless regulation, even without introducing the explicit benefits of preventing social disruption due to the lack of relevant legislation.

Lastly, we show that in the presence of both time-to-build and regulation, *not only uncertainty in time-to-build but also its presence can accelerate investment.* That is, under fixed-duration regulation, the state price of investment with fixed time-to-build can be higher than that without time-to-build. This result comes from the aforementioned argument: it is never optimal to invest under regulation if the project does not involve time-to-build, whereas it can be optimal to invest under regulation if there is time-to-build. This result is in sharp contrast with many existing studies on time-to-build. For instance, the seminal work of Majd and Pindyck (1987) assumed a maximum rate at which a firm can invest and showed that the presence of time-to-build delays investment. Some studies also report that uncertainty in price or demand can accelerate investment if the project involves time-to-build (e.g., Bar-Ilan and Strange (1996*a*,*b*)). However, they assume the firm's abandonment option, which truncates the downside risks of investment and forces stronger incentives to invest despite more uncertain demand. By contrast, this study focuses on the uncertainty in the timing of revenue generation and shows that not only uncertainty in time-to-build but also its presence can hasten investment, even without introducing the firm's option to truncate the downside risks.

The remainder of this paper is organized as follows. Section 2 reviews the literature on the investment-uncertainty relationship and the impacts of time-to-build and regulatory uncertainty on investment. Section 3 introduces the setup of the model, and Section 4 discusses the impacts of time-to-build on investment in the absence of regulation. Sections 4.1 and 4.2 consider the cases of fixed and uncertain time-to-build, respectively. Section 4.3 compares the two models, clarifying the effects of uncertainty in time-to-build on corporate investment. In Section 5, we discuss the impacts of regulation on investment in the absence of time-to-build. Sections 5.1 and 5.2 are dedicated to the cases of fixed- and uncertain-duration regulation, respectively. We compare these cases in Section 5.3, elucidating the effects of uncertainty in regulation on investment decision. Section 6 considers

[7] As noted in Leahy and Whited (1996), the option value of irreversible investment that represents the negative impact of uncertainty on investment can be read as the concavity of the marginal revenue product of capital.

the firm's investment decision in the presence of both time-to-build and regulation. Sections 6.1 and 6.2 discuss the cases in which both are fixed and uncertain, respectively. Section 6.3 sheds light on the effects of uncertainty in the timing of revenue generation by comparing the two models. Section 7 summarizes the main results and suggests future research. All proofs are presented in Appendix A. In the Online Appendix, we analyze the cases with fixed time-to-build and uncertain-duration regulation and vice versa, as well as the case with running costs incurred during time-to-build.

2 Literature review

The effects of uncertainty on investment have been discussed extensively in the corporate finance literature. Hartman (1972, 1973) and Abel (1983) showed that the convexity of marginal revenue product of capital, which is induced by the optimal adjustment of labor over time, makes uncertainty accelerate investment. However, McDonald and Siegel (1986) and Pindyck (1988, 1993) highlighted the irreversibility of investment and showed that the opportunity costs of investing now instead of waiting for more information make uncertainty delay investment.[8] Caballero (1991) demonstrated that the dominance between these two effects depends on the degree of market competition and the structure of adjustment costs. Abel and Eberly (1999) focused on the *long-run* effects of irreversible investment; while it reduces the capital stock in the short-run, it also prevents the firm from disinvesting, possibly resulting in a greater capital accumulation over time. They showed that these opposing effects lead to considerable ambiguity in investment-uncertainty relationship. Nakamura (1999) found that risk-aversion can induce a negative effect of uncertainty on investment even without introducing the irreversibility of investment. However, Saltari and Ticchi (2007) paid attention to the investment-uncertainty relationship in *aggregate* level and showed that risk aversion by itself is not enough to induce a negative relationship; it is positive even when agents are very risk averse as long as the elasticity of intertemporal substitution is low. These studies focus on the *uncertainty in output price or demand*, and little is known of the effects of *uncertainty in the timing of revenue generation* on corporate investment. Even if a firm accounts for uncertain demand when investing in a project, the timing at which it generates revenue is highly uncertain in many cases. It can be delayed significantly by time-to-build of the investment project or regulation that hinders the commercialization of new technologies or products.

Most empirical studies support the negative impacts of uncertainty on investment.[9] That is, the option value effects dominate the convexity effects. Ferderer (1993) employed the risk premium from the term structure of interest rates as a measure of uncertainty and found that it has significantly negative impacts on aggregate investment. Leahy and Whited (1996) used U.S. firm-level panel data, adopting the expected variance of the firms' daily stock return as a proxy of uncertainty, and found strong evidence of a negative impact of uncertainty on corporate investment. Guiso and Parigi (1999) adopted Italian firm-level data, using the subjective probability distribution of future demands from a survey conducted by the Bank of Italy to gauge uncertainty, and reported a negative investment-uncertainty relationship. Meinen and Roehe (2017) analyzed data from major European countries, testing five different proxies of uncertainty, and found pronounced negative impacts of uncertainty on investment.

Huizinga (1993) is one of a few studies that found a positive investment-uncertainty relationship. The author used U.S. data and showed that on aggregate level, the uncertainty of wage and price has a negative impact on investment, whereas that of profit rate has a positive impact. At the firm level, uncertainty of wage and input price is negatively associated with investment but that of output price is positively linked to investment. Bo and Lensin (2005) examined Dutch firm-level data and found a nonlinear relationship between uncertainty and investment; under low-level uncertainty, an increase in uncertainty raises investment, but for higher uncertainty, its increase reduces investment. Marmer and Slade (2018) analyzed the U.S. copper mining industry and their reduced-form estimation showed that an increase in uncertainty encourages investment in the presence of time-to-build. However, by estimating the structural model of Bar-Ilan and Strange (1996*a*), they found a nonlinear relationship between uncertainty and investment.

Majd and Pindyck (1987) pioneered the research on time-to-build in corporate finance by assuming a maximum

[8] Sarkar (2000) claimed that even though an increase in uncertainty raises the investment threshold, higher volatility can eventually increase the probability of hitting the threshold within a given time, and thus, the probability of investment.

[9] Carruth et al. (2000) provided a comprehensive review of empirical studies on investment-uncertainty relationship.

rate at which a firm can invest and showed that the presence of time-to-build delays investment. Bar-Ilan and Strange (1996*a*,*b*) explicitly modeled time-to-build by presuming that a certain amount of time must pass to generate revenue from a project and showed that uncertainty can hasten investment in the present of lags. This result, however, draws on the assumption that the firm has the option to abandon the project, which truncates the downside risk of the project. Bar-Ilan and Strange (1998) examined a project that requires a two-stage investment to generate revenue in the presence of time-to-build and demonstrated that the investment can be made sequentially if the firm has the option to suspend the follow-up investment. Pacheco-de-Almeida and Zemsky (2003) also investigated a multi-stage investment with time-to-build while accounting for competition. They reported that in a duopoly market, both firms choose to make either incremental or lumpy investments depending on the length of time-to-build. Bar-Ilan et al. (2002) solved an impulse control problem that minimizes the costs of excess capacity in the presence of time-to-build and showed that an increase in uncertainty can hasten the timing of investment but reduce its size when the lags are significantly long. These prior works, however, assumed a fixed time-to-build.

Some studies focus on *uncertainty* in time-to-build. Weeds (2002) examined research and development (R&D) competition in a duopoly market with an uncertain discovery time and showed that such uncertainty can delay investment despite the preemptive incentive in a winner-takes-all scenario. Alvarez and Keppo (2002) considered the case in which time-to-build increases with demand and demonstrated that this scenario creates a significant delay in investment. Jeon (2021*a*) examined the effects of uncertain time-to-build on a firm's investment and default decisions by considering debt financing and showed that the probability of default in the presence of time-to-build can be lower than that without the lags. Jeon (2021*b*) investigated duopolistic competition with asymmetric lengths of uncertain time-to-build and found that the dominated firm with longer lags can become a leader in the market by investing earlier than the dominant firm with shorter lags. Jeon (2023) studied a monopolistic firm's capital expansion with uncertain time-to-build and showed that both the initial and follow-investment can be made earlier in the presence of time-to-build than they would be without the lags, especially in a volatile market. These studies, however, did not compare the results to the case when time-to-build is known with certainty, leaving the sheer effects of uncertainty in time-to-build unexplored.

Koeva (2000) collected data regarding plant investment process and found that the average time-to-build is about two years in most industries and the lead time is not sensitive to business cycles. Zhou (2000) empirically showed that the presence of time-to-build can explain the positive correlation of investment. Salomon and Martin (2008) examined the semiconductor industry and reported that the length of time-to-build is associated with market competition, firm ownership, and firm/industry experience. Del Boca et al. (2008) analyzed Italian panel data and found strong evidence of a time-to-build effect on investment for structures but not for equipment. Tsoukalas (2011) showed that cash flows significantly influence a firm's investment decisions, even in a perfect capital market, if the project involves time-to-build. Hansen and Wagner (2017) examined the copper mining industry and identified the firm's incentive to retain sufficient cash when it takes time to generate cash flows. However, none of these studies empirically analyzed the effects of *uncertainty in time-to-build* on corporate investment.

Kalouptsidi (2014) is one of the very few studies to address the impact of *time-varying* time-to-build on investment empirically. The author analyzed data from the bulk shipping industry and compared the case of the endogenous time-varying time-to-build to counterfactual cases of constant and no time-to-build. the results revealed that moving from no to constant to time-varying time-to-build lowers both the level and volatility of investment and increases prices. However, the size of constant time-to-build is chosen as the minimum of the observations, rather than the mean or median. Thus, it is not the *uncertainty* of time-to-build but the *increase in its size* that induces the decrease of the level and volatility of investment. Oh and Yoon (2020) investigated the U.S. residential investment in the 2002-2011 housing boom-bust cycle and found that the increase in time-to-build during the boom is mainly due to construction bottlenecks, but that during the bust is driven by an increase in uncertainty.

In general, regulation is perceived to have a negative impact on investment and productivity, a view supported by many empirical studies. Alesina et al. (2005) used OECD data and showed that product market regulation has a negative and significant long-run effect on investment. In particular, they showed that deregulation, especially entry liberalization, positively affects investment. Klapper et al. (2006) analyzed European data and showed that market entry regulations hamper the creation of new firms and force new entrants to be larger. Barone and

Cingano (2011) found from OECD data that service regulation is negatively associated with the growth rate of value added, productivity, and exports. Fabrizio (2013) analyzed the impact of Renewable Portfolio Standard policies on the U.S. electricity industry and demonstrated that an increase in regulatory instability reduced new investment. Branstetter et al. (2014) investigated a regulatory reform in Portugal that reduced the cost of firm entry and found a positive impact on firm formation and employment. Specifically, they found that marginal firms that would have been most readily deterred by strict regulations were the largest beneficiaries of the reform. Gulen and Ion (2016) used a news-based index of policy uncertainty and found a substantial negative impact of policy and regulatory uncertainty on corporate investment, with a more significant impact on firms with a higher degree of investment irreversibility.

A few studies reported that regulation does not necessarily discourage investment. Marcus (1981) reviewed the literature on the impact of regulation on innovation and concluded that the impact is selective; that is, its impact is negative in some industries but it stimulates innovation in others by reducing risks. Djankov et al. (2002) analyzed data on the regulation of entry of start-up firms in 85 countries and found that stricter regulation is not associated with higher quality of goods; rather, it is related to higher corruption and larger unofficial economies, supporting the public choice theory over public interest theory. They, however, did not investigate the impact of regulation on investment. Hoffmann et al. (2009) showed that environmental and regulatory uncertainty does not necessarily discourage corporate investment and innovation. Engau and Hoffmann (2009) empirically studied firms' responses to the Kyoto Protocol and demonstrated that only a minority of firms actually delayed their investment decisions due to regulatory uncertainty. Lopez et al. (2017) distinguished regulatory uncertainty from regulation-induced uncertainty. They analyzed the impact of the European Union Emissions Trading System on firms' carbon abatement investment and showed that the former does not affect firms' investment decisions while the latter facilitates investment.

3 Setup

Suppose that a risk-neutral firm is considering an investment project with demand shocks that follow a geometric Brownian motion:

$$\mathrm{d}X_t = \mu X_t \mathrm{d}t + \sigma X_t \mathrm{d}W_t, \tag{1}$$

where μ and σ are positive constants and $(W_t)_{t\geq 0}$ is a standard Brownian motion on a filtered space $(\Omega, \mathcal{F}, \mathbb{F} := (\mathcal{F}_t)_{t\geq 0}, \mathbb{P})$ satisfying the usual conditions. For simplicity, we assume that the demand is price-inelasitic such that the monopolistic firm's revenue flow from this project is equal to X_t. The investment incurs lump-sum costs C and the variable costs of production are normalized to zero. The discount rate is $r(> \mu)$ to ensure finite value functions.

The project does not yield revenue immediately after the investment. Two factors hinder the firm's instant revenue generation. First, the project involves time-to-build such that revenue is generated only after a certain period elapses from the investment, possibly because of the R&D for new technologies or the installation of manufacturing facilities. Thus, this hindering factor is internal to the firm. As noted in the Introduction, time-to-build is prevalent in the real world, especially when the investment projects are based on state-of-the-art technologies or are of large-scale. Time-to-build can be either constant or random, denoted by T_B and τ_B, respectively. In the latter case, we assume it follows an exponential distribution with an intensity parameter λ_B; for simplicity, we assume τ_B is independent of W_t.

Next, the introduction of new technologies or products can be delayed due to regulation, which implies an external factor hindering the firm.[10] Technological innovation has been accelerating recently. As addressed in the Introduction, regulations increasingly do not keep up with its speed. To describe this hindrance to the firm's immediate revenue generation, we assume that the product can only be sold in the market after a certain time after the initial timing.[11] The duration of regulation, or equivalently, the timing of deregulation, can be either

[10]There are many different types of regulation, such as barriers to entry, rate-of-return regulation, price ceiling, minimum wage, and environmental regulation. In this study, we limit our analysis to a stringent regulation that prohibits the commercialization of goods or services until they are specified in legislation.

[11]Regulation in this strong sense can be generalized by assuming that the firm can raise a portion of revenue under regulation and starts to generate full revenue after the regulation is lifted.

constant or random, denoted by T_D and τ_D, respectively. In the latter case, we assume an exponential time with an intensity parameter λ_D, independent of both τ_B and W_t.

4 With time-to-build but no regulation

In this section, we assume there is no regulation regarding the investment project. That is, the technology underlying the project does not conflict with existing laws and institutions, and the only source that prevents the firm from raising instant revenue after the investment is time-to-build, which is internal to the firm (e.g., semiconductor plant building, power plant building, shipbuilding, mining natural resources, etc.). Specifically, we assume that time-to-build is fixed and uncertain in Sections 4.1 and 4.2, respectively, and compare the results in Section 4.3.

4.1 Fixed time-to-build

Suppose a constant time-to-build, denoted by T_B. The firm value before the investment can be written as follows:

$$V_{FN}(X) = \max_{T_{FN}\geq 0} \mathbb{E}\Big[\int_{\hat{T}_{FN}}^{\infty} e^{-rt}X_t \mathrm{d}t - e^{-rT_{FN}}C\Big|X_0 = X\Big]. \tag{2}$$

The investment timing can be characterized by the level of demand shock at which the firm invests. That is, $T_{FN} := \inf\{t > 0|X_t \geq X_{FN}\}$, where X_{FN} denotes the investment threshold with fixed time-to-build and no regulation. Note that though the firm invests at T_{FN}, it can raise revenue from $\hat{T}_{FN} := T_{FN} + T_B$, when the project is finished.

Following the standard arguments from the real options literature, we can derive the firm's optimal investment strategy and value function as follows:

Proposition 1 (Fixed time-to-build and no regulation) *Given demand shock X, the firm value with fixed time-to-build T_B in the absence of regulation is*

$$V_{FN}(X) = \begin{cases} \Big[\frac{X_{FN}e^{-(r-\mu)T_B}}{r-\mu} - C\Big]\Big(\frac{X}{X_{FN}}\Big)^{\beta}, & \text{if } X < X_{FN}, \\ \frac{Xe^{-(r-\mu)T_B}}{r-\mu} - C, & \text{if } X \geq X_{FN}, \end{cases} \tag{3}$$

where the optimal investment threshold is

$$X_{FN} = \frac{\beta(r-\mu)Ce^{(r-\mu)T_B}}{\beta - 1}, \tag{4}$$

and

$$\beta := \frac{1}{2} - \frac{\mu}{\sigma^2} + \sqrt{\Big(\frac{1}{2} - \frac{\mu}{\sigma^2}\Big)^2 + \frac{2r}{\sigma^2}} \ (> 1). \tag{5}$$

Proof See Appendix A.1.

One can easily see that the firm value and investment threshold in (3) and (4) converge to those from a standard real options model as $T_B \to 0$.

4.2 Uncertain time-to-build

Now let us suppose that time-to-build is a random variable, denoted by τ_B, and follows an exponential distribution with parameter λ_B. The firm value before investment is

$$V_{UN}(X) = \max_{T_{UN}\geq 0} \mathbb{E}\Big[\int_{\hat{T}_{UN}}^{\infty} e^{-rt}X_t \mathrm{d}t - e^{-rT_{UN}}C\Big|X_0 = X\Big], \tag{6}$$

where the investment timing is $T_{UN} := \inf\{t > 0|X_t \geq X_{UN}\}$ and X_{UN} denotes the investment threshold with uncertain time-to-build and no regulation. While the investment occurs at T_{UN}, the firm starts to make revenue from $\hat{T}_{UN} := T_{UN} + \tau_B$, which is uncertain, unlike $\hat{T}_{FN}$ in (2). Following similar arguments, we can derive the firm's optimal investment strategy and value function as follows:

Proposition 2 (Uncertain time-to-build and no regulation) *Given demand shock X, the firm value with uncertain time-to-build τ_B in the absence of regulation is*

$$V_{UN}(X) = \begin{cases} \left[\frac{\lambda_B X_{UN}}{(r-\mu)(r+\lambda_B-\mu)} - C\right]\left(\frac{X}{X_{UN}}\right)^{\beta}, & \text{if } X < X_{UN}, \\ \frac{\lambda_B X}{(r-\mu)(r+\lambda_B-\mu)} - C, & \text{if } X \geq X_{UN}, \end{cases} \tag{7}$$

where the optimal investment threshold is

$$X_{UN} = \frac{\beta(r-\mu)(r+\lambda_B-\mu)C}{(\beta-1)\lambda_B}. \tag{8}$$

Proof See Appendix A.2.

It is straightforward that the firm value and investment threshold in (7) and (8) converge to those from a standard real options model as $\lambda_B \to \infty$.

4.3 Model comparison and discussion

Comparing Propositions 1 and 2, we can obtain the following result:

Proposition 3 (Uncertainty in time-to-build) *In the absence of regulation, uncertainty in time-to-build always accelerates investment and enhances firm value:*[12]

$$X_{UN} \leq X_{FN} \quad \text{and} \quad V_{UN}(X) \geq V_{FN}(X) \quad \text{for all } T_B = 1/\lambda_B \in (0, \infty). \tag{9}$$

Proof See Appendix A.3.

The economic rationale behind this counterintuitive result is as follows. The discount factor $e^{-r\tau_B}$ is strictly convex with respect to τ_B, and thus, Jensen's inequality ensures that $\mathbb{E}[e^{-r\tau_B}] \geq e^{-r\mathbb{E}[\tau_B]} = e^{-rT_B}$ always holds for all $T_B = 1/\lambda_B \in (0, \infty)$.[13] In other words, the firm discounts cash flows less if the timing at which they are generated is uncertain than the case in which the timing is known with certainty, provided that the size of time-to-build is identical in terms of expectation. Intuitively, this is because *good news is discounted less and bad news is discounted more.* When the investment project is finished earlier than expected (i.e., $\tau_B < T_B$), the gains from this surprise are discounted over a relatively short period. When it takes longer than expected to finish the project (i.e., $\tau_B > T_B$), the extra losses are discounted over a longer period. Note that the asymmetric effects of good and bad news on firm value are unrelated to risk aversion or negativity bias. It is the *uncertainty in timing* that yields the asymmetric effects of good and bad news based on rational expectations. This result is robust in that the same result holds even when the project incurs running costs until its completion, which is discussed in detail in the Online Appendix.

The positive impacts of uncertainty in time-to-build on investment and firm value are also robust in that they do not depend on a specific distribution for describing the uncertainty. Specifically, Proposition 3 can be generalized as follows:

Proposition 4 (Generalized uncertainty in time-to-build) *Suppose time-to-build τ_B^F and τ_B^G have cumulative distribution functions F and G, respectively, and τ_B^G is a mean-preserving spread of τ_B^F. In the absence of regulation, uncertainty in time-build always accelerates investment and enhances firm value:*

$$X_{UN}^G \leq X_{UN}^F \quad \text{and} \quad V_{UN}^G(X) \geq V_{UN}^F(X), \tag{10}$$

where X_{UN}^i and $V_{UN}^i(X)$ denote the optimal investment threshold and firm value, respectively, with time-to-build τ_B^i for $i \in \{F, G\}$.

[12] Note that this result holds up until the firm makes the investment (i.e., for $t \in [0, T_{UN}(\leq T_{FN}))$). After the investment, the remaining fixed time-to-build decreases as time passes, while the expected uncertain time-to-build remains the same due to the memoryless property of the exponential distribution. Thus, the dominance of firm value with uncertain time-to-build over that with fixed time-to-build cannot be guaranteed after the investment.

[13] Note that this argument holds for τ_B following *any* distribution, provided that it is a mean-preserving spread of T_B (i.e., $\mathbb{E}[\tau_B] = T_B$).

PROOF See Appendix A.4.

Extant studies on the effects of uncertainty on investment mostly focused on *uncertainty in output price or demand.* For instance, Hartman (1972, 1973) and Abel (1983) showed that uncertainty accelerates investment because of the convexity of the marginal profitability of capital, whereas McDonald and Siegel (1986) and Pindyck (1988, 1993) showed that uncertainty delays investment because of the opportunity cost of investing now rather than waiting for more information. By contrast, our study clarifies the effects of *uncertainty in time-to-build*, which we presume to be independent of uncertainty in demand, and shows that *it always accelerates investment and enhances firm value, regardless of the distribution* that describes the uncertainty in time-to-build. We obtain this novel result by shedding light on *uncertainty in the time dimension instead of that in state space.*

Most empirical studies report a negative impact of uncertainty on corporate investment,[14] but a few studies found a positive or nonlinear investment-uncertainty relationship. For instance, Huizinga (1993) analyzed the U.S. manufacturing industry and found that on aggregate level, the uncertainty of the profit rate has a positive effect; at the firm-level, uncertainty of output price is positively linked to investment. Bo and Lensin (2005) investigated Dutch firm-level data and found an inverted U-shaped relationship between uncertainty and investment; for low level of uncertainty, an increase of uncertainty accelerates investment, but for higher uncertainty, its increase reduces investment. Marmer and Slade (2018) examined the U.S. copper mining industry and found that an increase in uncertainty accelerates investment in the presence of time-to-build. Although not explicitly modeled in these studies, the positive investment-uncertainty relationship might be partly due to uncertainty in time-to-build; this hypothesis, for which there is not yet evidence, deserves to be examined with empirical data in the future.

Following the same arguments in Propositions 1 and 2, we can evaluate the state prices of investment in the absence of regulation as follows:

Corollary 1 (State prices of investment in the absence of regulation) *Given demand shock X, the state prices of investment in the absence of regulation with fixed and uncertain time-to-build are*

$$\phi_{FN}(X) := \mathbb{E}\big[e^{-rT_{FN}}\big|X_0 = X\big] = \Big(\frac{X}{X_{FN}}\Big)^{\beta}, \tag{11}$$

$$\phi_{UN}(X) := \mathbb{E}\big[e^{-rT_{UN}}\big|X_0 = X\big] = \Big(\frac{X}{X_{UN}}\Big)^{\beta}, \tag{12}$$

respectively.

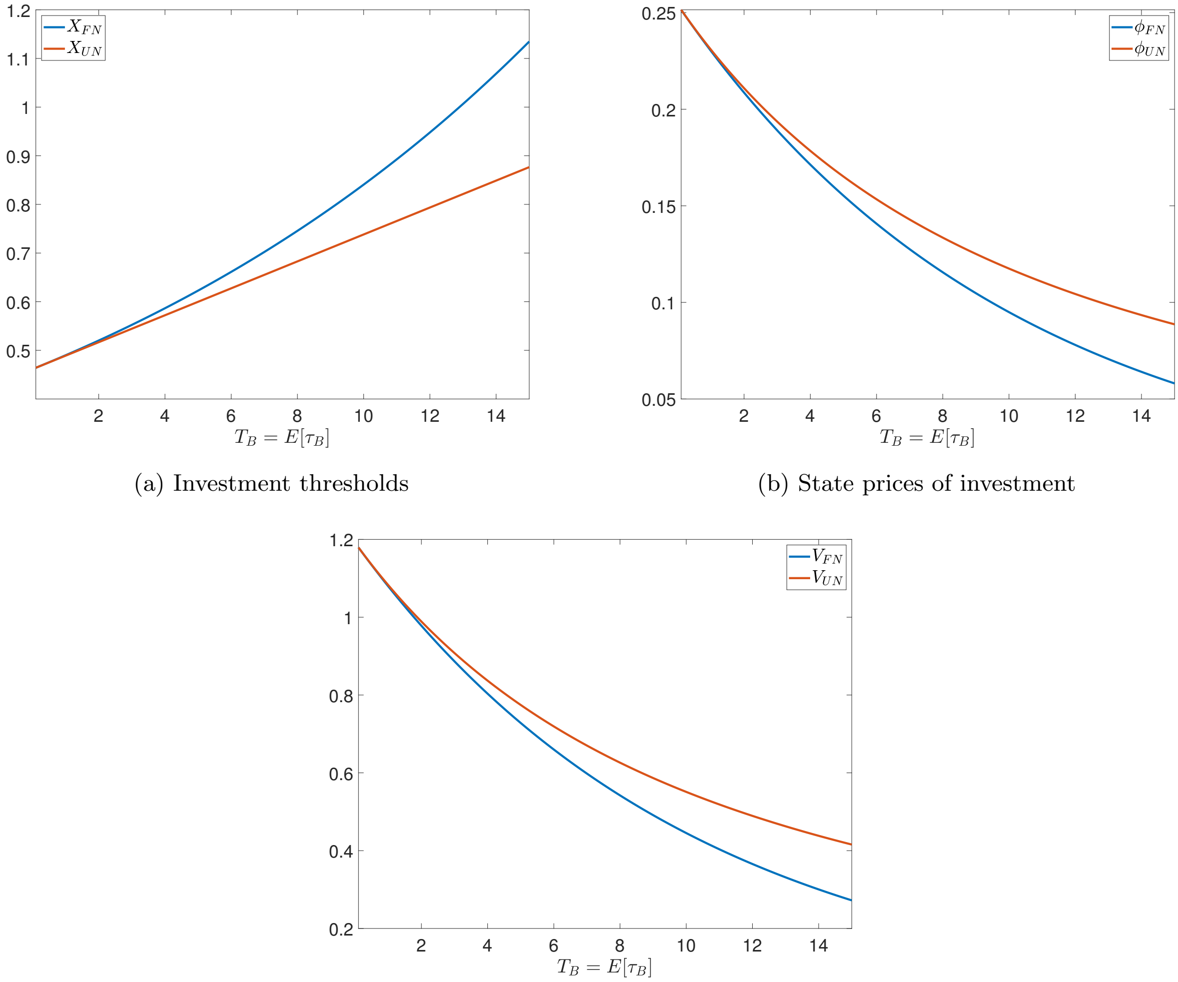
PROOF See Appendix A.5.

Figure 1 presents the results of comparative statics regarding the size of time-to-build (i.e., $T_B = 1/\lambda_B$) based on the parameters in Table 1. Figure 1a clearly shows that *the presence of time-to-build* delays investment, whether it is fixed or uncertain. That is, both X_{FN} and X_{UN} strictly increase with $T_B = 1/\lambda_B$. This is natural considering that the expected profits from the investment strictly decrease with the length of time-to-build. However, *uncertainty in time-to-build* accelerates investment compared to the case of time-to-build with certainty (i.e., $X_{UN} < X_{FN}$), provided that the size of time-to-build is identical in terms of expectations. This naturally leads to the dominance of state price of investment, and thus, the dominance of firm value in the former case over that in the latter case (i.e., $\phi_{UN} > \phi_{FN}$ and $V_{UN} > V_{FN}$ in Figures 1b and 1c, respectively).

Notation	Value	Description
r	0.08	Risk-free rate
μ	0.02	Expected growth rate of demand shock
σ	0.3	Volatility of demand shock
C	3	Lump-sum investment costs
X	0.2	Initial demand shock

Table 1: Benchmark parameters for numerical calculation

[14]See Ferderer (1993), Leahy and Whited (1996), Guiso and Parigi (1999), and Meinen and Roehe (2017).



(a) Investment thresholds

(b) State prices of investment

(c) Initial firm values

Figure 1: Comparative statics with respect to the size of time-to-build in the absence of regulation

5 With regulation but no time-to-build

In this section, we assume that the investment project does not involve investment lags. That is, the technology underlying the project is already in place and its commercialization does not take much time. In this case, only regulation, an external factor, prevents the firm from generating revenue after the investment (e.g., cannabis cultivation,[15] euthanasia assistance,[16] commercialization of creations generated by artificial intelligence (AI)[17]).

In this case, even if the firm invests under regulation, it can start to raise revenue after the regulation is lifted. Thus, we can obtain the following intuitive result:

Lemma 1 *In the absence of time-to-build, it is never optimal for the firm to invest under regulation, whether the timing of deregulation is certain or not.*

Proof See Appendix A.6.

[15] Although the use of cannabis has been increasingly legalized or decriminalized in recent years, its recreational use is still illegal in most countries; even its medical use is illegal in many of them. As of 2023, the commercial sale of recreational cannabis is legalized nationwide in only three countries: Canada, Thailand, and Uruguay.

[16] As of 2023, active voluntary euthanasia is legal in only nine countries: Australia, Belgium, Canada, Colombia, Luxembourg, Netherlands, New Zealand, Portugal, and Spain.

[17] Although AI-generated works have been increasingly commercialized in recent years, they face significant risks of legal disputes. In the U.S., works created solely by AI cannot be protected by copyright because it lacks *human* authorship, as stipulated by Chapter 306 of the Compendium of U.S. Copyright Office Practices. In 2020, the European Commission proposed a four-step test as a guildeline for AI-generated creations to qualify as works, with one criterion being *human* intellectual effort.

We assume that the duration of regulation, or equivalently, the timing of deregulation, is fixed and uncertain in Sections 5.1 and 5.2, respectively, and compare the results in Section 5.3.

5.1 Fixed deregulation timing

Let us assume that the duration of regulation is known as a constant T_D.[18] By backward induction, suppose that the regulation has already been lifted (i.e., $t \geq T_D$). The firm value after deregulation can be expressed as follows:

$$V_{NF}^D(X) = \max_{T_{NF} \geq 0} \mathbb{E}\Big[\int_{T_{NF}}^{\infty} e^{-rt} X_t \mathrm{d}t - e^{-rT_{NF}} C \Big| X_0 = X\Big], \tag{13}$$

where the investment timing is $T_{NF} := \inf\{t \geq T_D | X_t \geq X_{NF}\}$ and X_{NF} denotes the investment threshold with no time-to-build and fixed deregulation timing. Following similar arguments as in Section 4.1, the firm value after deregulation is obtained as follows:

$$V_{NF}^D(X) = \begin{cases} \Big(\frac{X_{NF}}{r-\mu} - C\Big)\Big(\frac{X}{X_{NF}}\Big)^{\beta}, & \text{if } X < X_{NF}, \\ \frac{X}{r-\mu} - C, & \text{if } X \geq X_{NF}, \end{cases} \tag{14}$$

where the optimal investment threshold is

$$X_{NF} = \frac{\beta(r-\mu)C}{\beta - 1}. \tag{15}$$

Now, let us suppose that the regulation has not been lifted yet (i.e., $t < T_D$). As addressed in Lemma 1, the firm never invests under regulation. In other words, it is possible that the investment is made as soon as the regulation is lifted. This occurs when the demand by the time the regulation is lifted is equal to or greater than the investment threshold (i.e., $X_{T_D} \geq X_{NF}$). In this case, the firm's option to invest can be read as a European option with maturity T_D. If the demand at the timing of deregulation falls short of the investment threshold (i.e., $X_{T_D} < X_{NF}$), the firm's option is an American option that will be exercised at the first-hitting time of the investment threshold thereafter (i.e., T_{NF}).

Given these arguments, the firm value at the initial timing under regulation is[19]

$$\begin{aligned} V_{NF}^R(X) &= \mathbb{E}\big[e^{-rT_D} V_{NF}^D(X_{T_D}) \big| X_0 = X\big] \\ &= \mathbb{E}\Big[\mathbf{1}_{\{X_{T_D} \geq X_{NF}\}} e^{-rT_D}\Big(\frac{X_{T_D}}{r-\mu} - C\Big) + \mathbf{1}_{\{X_{T_D} < X_{NF}\}} e^{-rT_D}\Big(\frac{X_{NF}}{r-\mu} - C\Big)\Big(\frac{X_{T_D}}{X_{NF}}\Big)^{\beta} \Big| X_0 = X\Big]. \end{aligned} \tag{16}$$

Note that the first term on the right-hand side of (16) corresponds to the case in which the European call option with maturity T_D and strike price X_{NF} ends up being in-the-money (i.e., $X_{T_D} \geq X_{NF}$), although its payoff is not in the exact form of a traditional European-style financial option. The second term shows that if the European call option ends up being out-of-the-money (i.e., $X_{T_D} < X_{NF}$), the firm is given an American call option that it can exercise at any time thereafter.

The firm value before deregulation described in (16) can be evaluated in a closed-form as follows:[20]

Proposition 5 (No time-to-build and fixed deregulation timing) *Given demand shock X, the firm value at the initial timing under regulation with fixed deregulation timing T_D in the absence of time-to-build is*

$$V_{NF}^R(X) = \frac{Xe^{-(r-\mu)T_D}}{r-\mu} N(d_1) - e^{-rT_D} C N(d_2) + \Big(\frac{X_{NF}}{r-\mu} - C\Big)\Big(\frac{X}{X_{NF}}\Big)^{\beta} N(d_3), \tag{17}$$

where

$$d_1 := \frac{\ln \frac{X}{X_{NF}} + (\mu + \frac{\sigma^2}{2})T_D}{\sigma\sqrt{T_D}}, \tag{18}$$

$$d_2 := d_1 - \sigma\sqrt{T_D}, \tag{19}$$

$$d_3 := -d_2 - \beta\sigma\sqrt{T_D}, \tag{20}$$

and $N(\cdot)$ denotes the cumulative distribution function of standard normal distribution.

[18]For instance, a term of patents corresponds to the regulation with a fixed duration. Before a patent's expiration, firms other than the patentee cannot commercialize the patented technology (unless they risk a legal dispute over patent infringement). In the U.S. and Europe, the term of a patent is generally 20 years from the filing date.

[19]For $t \in (0, T_D)$, the firm value is $\mathbb{E}[e^{-(T_D-t)} V_{NF}^D(X_{T_D}) | X_t = X]$.

[20]For $t \in (0, T_D)$, the firm value can be evaluated as (17) through (20) with $T_D - t$ instead of T_D.

PROOF See Appendix A.7.

The first two terms and the last term on the right-hand side of (17) correspond to the first and second terms on the right-hand side of (16) (i.e., the cases with $X_{T_D} \geq X_{NF}$ and $X_{T_D} < X_{NF}$), respectively. Note that for $X < X_{NF}$, $\lim_{T_D \to 0} d_1 = -\infty$, $\lim_{T_D \to 0} d_2 = -\infty$, and $\lim_{T_D \to 0} d_3 = \infty$ hold, and thus, $V_{NF}^R(X)$ in (17) converges to $V_{NF}^D(X)$ in the upper case of (14) as $T_D \to 0$. For $X \geq X_{NF}$, $\lim_{T_D \to 0} d_1 = \infty$, $\lim_{T_D \to 0} d_2 = \infty$, and $\lim_{T_D \to 0} d_3 = -\infty$, which also amounts to the convergence of $V_{NF}^R(X)$ in (17) to $V_{NF}^D(X)$ in the lower case of (14).

5.2 Uncertain deregulation timing

We now assume that the duration of regulation is uncertain such that the timing of deregulation τ_D follows an exponential distribution with parameter λ_D. The firm value after deregulation (i.e., $t \geq \tau_D$) can be described as

$$V_{NU}^D(X) = \max_{T_{NU} \geq 0} \mathbb{E}\Big[\int_{T_{NU}}^{\infty} e^{-rt} X_t \mathrm{d}t - e^{-rT_{NU}} C \Big| X_0 = X\Big], \tag{21}$$

where $T_{NU} := \inf\{t \geq \tau_D | X_t \geq X_{NU}\}$ denotes the investment timing and X_{NU} denotes the investment threshold with no time-to-build and uncertain deregulation timing. Note that (21) is essentially equivalent to $V_{NF}^D(X)$ in (13). Thus, the firm value $V_{NU}^D(X)$ and the investment threshold X_{NU} are identical to (14) and (15), respectively.

As noted in Lemma 1, the firm never invests under regulation if the project involves no time-to-build, even under uncertain-duration regulation. In other words, the firm value under regulation switches to (21) at the timing of deregulation. As discussed in Section 5.1, deregulation can trigger immediate investment (i.e., $X_{\tau_D} \geq X_{NU}$) or yield a standard real option exercised at the first-hitting time of the investment threshold (i.e., $X_{\tau_D} < X_{NU}$). Thus, the firm value under regulation (i.e., $t < \tau_D$) can be described as follows:[21]

$$\begin{aligned} V_{NU}^R(X) &= \mathbb{E}\big[e^{-r\tau_D} V_{NU}^D(X_{\tau_D}) \big| X_0 = X\big] \\ &= \mathbb{E}\Big[\mathbf{1}_{\{X_{\tau_D} \geq X_{NU}\}} e^{-r\tau_D}\Big(\frac{X_{\tau_D}}{r-\mu} - C\Big) + \mathbf{1}_{\{X_{\tau_D} < X_{NU}\}} e^{-r\tau_D}\Big(\frac{X_{NU}}{r-\mu} - C\Big)\Big(\frac{X_{\tau_D}}{X_{NU}}\Big)^{\beta} \Big| X_0 = X\Big]. \end{aligned} \tag{22}$$

Note that the only difference between (16) and (22) is that the timing of deregulation is fixed at T_D in (16) and is an exponential time τ_D in (22). That is, the firm's option to invest is exercised immediately if it is in-the-money at the exponential time τ_D (i.e., $X_{\tau_D} \geq X_{NU}$). If out-of-the-money at the time of deregulation (i.e., $X_{\tau_D} < X_{NU}$), the firm's option is an American option exercised at the first-hitting time of the investment threshold.

The firm value described in (22) can be evaluated in a closed-form as follows:

Proposition 6 (No time-to-build and uncertain deregulation timing) *Given demand shock X, the firm value under regulation with uncertain deregulation timing τ_D in the absence of time-to-build is*

$$V_{NU}^R(X) = \begin{cases} \Big(\frac{X_{NU}}{r-\mu} - C\Big)\Big\{\Big(\frac{X}{X_{NU}}\Big)^{\beta} - \frac{\beta-\gamma_D}{\beta_D-\gamma_D}\Big(\frac{X}{X_{NU}}\Big)^{\beta_D}\Big\} \\ \quad + \frac{1}{\beta_D-\gamma_D}\Big[\frac{(1-\gamma_D)\lambda_D X_{NU}}{(r-\mu)(r+\lambda_D-\mu)} + \frac{\gamma_D\lambda_D C}{r+\lambda_D}\Big]\Big(\frac{X}{X_{NU}}\Big)^{\beta_D}, & \text{if } X < X_{NU}, \\ \frac{\lambda_D X}{(r-\mu)(r+\lambda_D-\mu)} - \frac{\lambda_D C}{r+\lambda_D} + \frac{1}{\beta_D-\gamma_D}\Big[\frac{(\beta_D-\beta)X_{NU}}{r-\mu} - (\beta_D-\beta)C \\ \quad - \Big\{\frac{(\beta_D-1)\lambda_D X_{NU}}{(r-\mu)(r+\lambda_D-\mu)} - \frac{\beta_D\lambda_D C}{r+\lambda_D}\Big\}\Big]\Big(\frac{X}{X_{NU}}\Big)^{\gamma_D}, & \text{if } X \geq X_{NU}, \end{cases} \tag{23}$$

where

$$\beta_D := \frac{1}{2} - \frac{\mu}{\sigma^2} + \sqrt{\Big(\frac{1}{2} - \frac{\mu}{\sigma^2}\Big)^2 + \frac{2(r+\lambda_D)}{\sigma^2}} \ (>1), \tag{24}$$

$$\gamma_D := \frac{1}{2} - \frac{\mu}{\sigma^2} - \sqrt{\Big(\frac{1}{2} - \frac{\mu}{\sigma^2}\Big)^2 + \frac{2(r+\lambda_D)}{\sigma^2}} \ (<0). \tag{25}$$

PROOF See Appendix A.8.

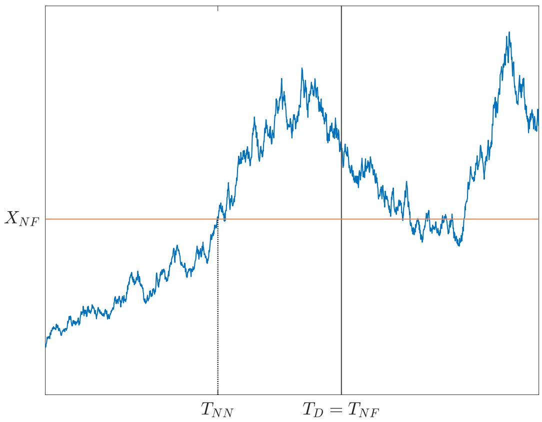
The first row of the upper case in (23) corresponds to the second term on the right-hand side of (22) (i.e., $X_{\tau_D} < X_{NU}$), while the second row of the upper case in (23) corresponds to the first term of (22) (i.e., $X_{\tau_D} \geq X_{NU}$). It is straightforward to see that $V_{NU}^R(X)$ in (23) converges to $V_{NU}^D(X)$, which is identical to $V_{NF}^D(X)$ in (14), as $\lambda_D \to \infty$, whether $X < X_{NU}$ or $X \geq X_{NU}$.

[21]Given demand shock X, the firm value under regulation is the same as (22) for all $t \in [0, \tau_D)$ because of the memoryless property of the exponential distribution.

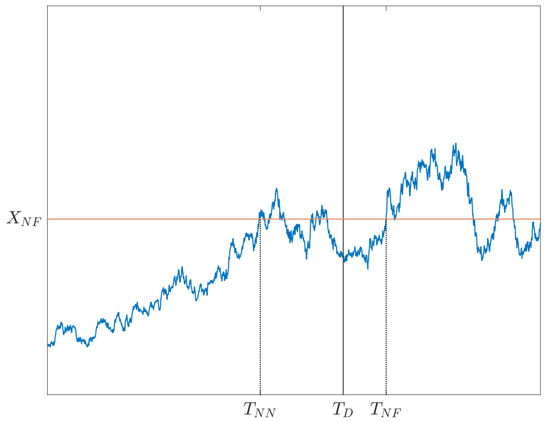
5.3 Model comparison and discussion

Recall that the firm never invests under regulation if the investment project does not involve time-to-build, whether the duration of regulation is certain or not (Lemma 1). Thus, investment triggered by deregulation (i.e., $X_{T_D} \geq X_{NF}$ and $X_{\tau_D} \geq X_{NU}$) implies that the firm would have already invested had it not been for the regulation. In other words, it implies that *the investment has been distorted by the presence of regulation.*

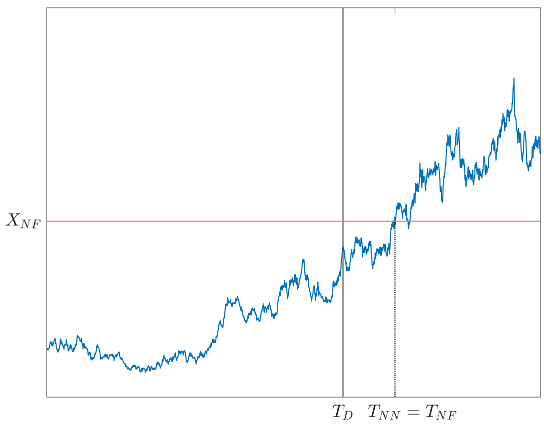
However, this is only half the story. Even when the investment is not triggered by deregulation (i.e., $X_{T_D} < X_{NF}$ and $X_{\tau_D} < X_{NU}$), it could have been delayed by the regulation. More specifically, the demand shocks can reach the investment threshold under regulation and decrease below the threshold before deregulation, hitting it again after deregulation. That is, even when $T_{NF} = \inf\{t \geq T_D | X_t \geq X_{NF}\}$ and $T_{NU} = \inf\{t \geq \tau_D | X_t \geq X_{NU}\}$ are not equal to T_D and τ_D, respectively, they do not necessarily coincide with $T_{NN} := \inf\{t > 0 | X_t \geq X_{NF} (= X_{NU})\}$, the first-hitting time of the investment threshold. In such cases, the investment decision is distorted by regulation, although not triggered by deregulation.



(a) Investment delayed by regulation and triggered by deregulation



(b) Investment delayed by regulation but not triggered by deregulation



(c) Investment not affected by regulation

Figure 2: Possible scenarios of investment in the presence of regulation with fixed duration

Figure 2 describes possible scenarios of investment in the presence of fixed-duration regulation (i.e., $T_D = 3$) based on the parameters in Table 1. Figures 2a and 2b illustrate the investment distorted by the presence of regulation; the former is triggered by deregulation (i.e., $T_{NN} < T_D = T_{NF}$), while the latter is not triggered by deregulation but is delayed by the presence of regulation (i.e., $T_{NN} < T_D < T_{NF}$). Namely, investment is distorted by regulation if the demand shocks reach the investment threshold before the regulation is lifted (i.e., $T_{NN} < T_D$), whether it is triggered by deregulation or not. If the demand shocks have not reached the investment threshold until the regulation is lifted (i.e., $T_D < T_{NN} = T_{NF}$), the presence of regulation does not affect the investment (Figure 2c). These arguments hold even when the duration of regulation is uncertain, and we omit the repetitive exposition and graphical illustration for brevity.

Investment triggered by deregulation is in line with empirical evidence. Alesina et al. (2005) analyzed OECD data and found that deregulation, especially entry liberalization, positively affects investment. Branstetter et al. (2014) examined a regulatory reform in Portugal and found that deregulation had a positive impact on firm formation and employment. In particular, they found that the positive effects were the greatest for marginal firms that would have been most readily deterred by strict regulations; investment triggered at the timing of deregulation corresponds to this case.

With these arguments, we can obtain the following result:

Proposition 7 (Uncertainty in regulation) *In the absence of time-to-build, uncertainty in regulation mitigates (and aggravates) the distortion in the firm's investment decision induced by regulation if* $\Gamma_F := \hat{\Gamma}_F + \bar{\Gamma}_F >$ *(and* $<$*)* $\Gamma_U := \hat{\Gamma}_U + \bar{\Gamma}_U$ *holds where*

$$\hat{\Gamma}_F := \mathbb{P}(X_{T_D} \geq X_{NF}|X_0 = X) = N(d_2), \tag{26}$$

$$\bar{\Gamma}_F := \mathbb{P}(X_{T_D} < X_{NF}|X_0 = X) - \mathbb{P}(\max_{t\in[0,T_D]} X_t < X_{NF}|X_0 = X) = \Big(\frac{X}{X_{NF}}\Big)^{1-2\mu/\sigma^2} N(\bar{d}_2), \tag{27}$$

$$\hat{\Gamma}_U := \mathbb{P}(X_{\tau_D} \geq X_{NU}|X_0 = X) = \frac{-\hat{\gamma}_D}{\hat{\beta}_D - \hat{\gamma}_D}\Big(\frac{X}{X_{NU}}\Big)^{\hat{\beta}_D}, \tag{28}$$

$$\bar{\Gamma}_U := \mathbb{P}(X_{\tau_D} < X_{NU}|X_0 = X) - \mathbb{P}(\max_{t\in[0,\tau_D]} X_t < X_{NU}|X_0 = X) = \Big(1 + \frac{\hat{\gamma}_D}{\hat{\beta}_D - \hat{\gamma}_D}\Big)\Big(\frac{X}{X_{NU}}\Big)^{\hat{\beta}_D}, \tag{29}$$

with

$$\bar{d}_2 := d_2 - (2\mu/\sigma - \sigma)\sqrt{T_D}, \tag{30}$$

and

$$\hat{\beta}_D := \frac{1}{2} - \frac{\mu}{\sigma^2} + \sqrt{\Big(\frac{1}{2} - \frac{\mu}{\sigma^2}\Big)^2 + \frac{2\lambda_D}{\sigma^2}} \ (> 1), \tag{31}$$

$$\hat{\gamma}_D := \frac{1}{2} - \frac{\mu}{\sigma^2} - \sqrt{\Big(\frac{1}{2} - \frac{\mu}{\sigma^2}\Big)^2 + \frac{2\lambda_D}{\sigma^2}} \ (< 0). \tag{32}$$

Proof See Appendix A.9.

Note that $\hat{\Gamma}_F$ and $\hat{\Gamma}_U$ are the probabilities of investment triggered by deregulation, with fixed and uncertain timing, respectively (Figure 2a), while $\bar{\Gamma}_F$ and $\bar{\Gamma}_U$ are those not triggered by deregulation but delayed by the presence of regulation with certainty and uncertainty, respectively (Figure 2b). Clearly, the probabilities of investment not affected by regulation with certainty and uncertainty are $1 - \Gamma_F$ and $1 - \Gamma_U$, respectively (Figure 2c).

From Proposition 7, we can also derive the following result:

Corollary 2 (Duration of regulation and investment distortion) *When regulation is known with certainty, the probability of investment distortion by the regulation increases with its duration if*

$$\frac{\ln(X/X_{NF})}{T_D} < \mu - \frac{\sigma^2}{2} < \frac{\ln(X_{NF}/X)}{T_D}. \tag{33}$$

When regulation is uncertain, the probability of investment distortion by the regulation always increases with its expected duration.

Proof See Appendix A.10.

We must also consider not only the *likelihood* of investment distortion by regulation but also its *magnitude*. That is, we should consider the expected delay of investment by regulation. To fully evaluate the impact of regulation on the investment decision, we must calculate the state prices of investment. Following similar arguments as in Propositions 5 and 6, we can derive these values as follows:

Corollary 3 (State prices of investment in the absence of time-to-build) *Given demand shock X, the state price of investment in the absence of time-to-build with fixed deregulation timing is*

$$\phi_{NF}(X) := \mathbb{E}\big[e^{-rT_{NF}}\big|X_0 = X\big] = \bar{\phi}_{NF}(X) + \hat{\phi}_{NF}(X), \tag{34}$$

where

$$\hat{\phi}_{NF}(X) := \mathbb{E}\big[\mathbf{1}_{\{X_{T_D} \geq X_{NF}\}} e^{-rT_D}\big|X_0 = X\big] = e^{-rT_D} N(d_2), \tag{35}$$

$$\bar{\phi}_{NF}(X) := \mathbb{E}\big[\mathbf{1}_{\{X_{T_D} < X_{NF}\}} e^{-rT_{NF}}\big|X_0 = X\big] = \Big(\frac{X}{X_{NF}}\Big)^{\beta} N(d_3), \tag{36}$$

denote the state prices of investment triggered at the deregulation timing and the first-hitting time of the investment threshold after deregulation, respectively.

Similarly, the state price of investment in the absence of time-to-build with uncertain deregulation timing is

$$\phi_{NU}(X) := \mathbb{E}\big[e^{-rT_{NU}}\big|X_0 = X\big] = \hat{\phi}_{NU}(X) + \bar{\phi}_{NU}(X), \tag{37}$$

where

$$\hat{\phi}_{NU}(X) := \mathbb{E}\big[\mathbf{1}_{\{X_{\tau_D} \geq X_{NU}\}} e^{-r\tau_D}\big|X_0 = X\big] = -\frac{\lambda_D \gamma_D}{(r+\lambda_D)(\beta_D - \gamma_D)}\Big(\frac{X}{X_{NU}}\Big)^{\beta_D}, \tag{38}$$

$$\bar{\phi}_{NU}(X) := \mathbb{E}\big[\mathbf{1}_{\{X_{\tau_D} < X_{NU}\}} e^{-rT_{NU}}\big|X_0 = X\big] = \Big(\frac{X}{X_{NU}}\Big)^{\beta} - \frac{\beta - \gamma_D}{\beta_D - \gamma_D}\Big(\frac{X}{X_{NU}}\Big)^{\beta_D}, \tag{39}$$

denote the state prices of investment triggered at the deregulation timing and the first-hitting time of the investment threshold after deregulation, respectively.

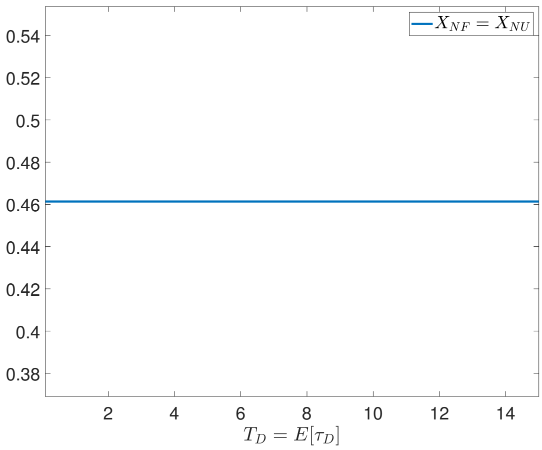
Proof See Appendix A.11.

It is straightforward that in the absence of regulation and time-to-build, the firm invests at T_{NN}, the first-hitting time of the investment threshold $X_{NF}(= X_{NU})$, and the state price of investment is $\phi_{NN}(X) := \mathbb{E}[e^{-rT_{NN}}|X_0 = X] = (X/X_{NF})^{\beta}$. Thus, we can measure the investment distortion by regulation with fixed and uncertain duration as $\tilde{\phi}_{NF} := \phi_{NN} - \phi_{NF}$ and $\tilde{\phi}_{NU} := \phi_{NN} - \phi_{NU}$, respectively. Note that these measures consider not only the *likelihood* of investment distortion by regulation but also its *magnitude.*

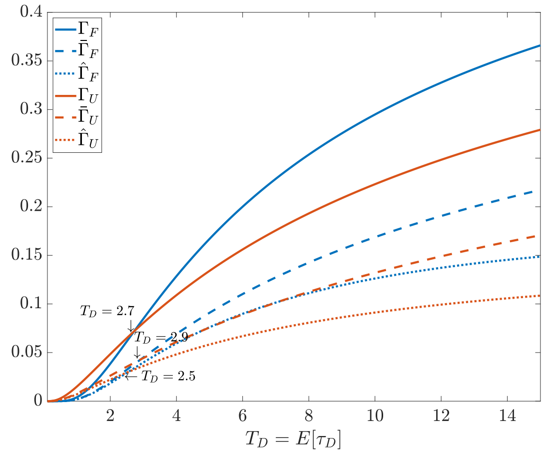
Figure 3 presents the comparative statics regarding the duration of regulation (i.e., $T_D = 1/\lambda_D$) based on the parameters in Table 1. It is obvious that the investment threshold after deregulation is independent of the duration of regulation, whether it is certain or not (Figure 3a).

Figure 3b illustrates the likelihood of investment distorted by regulation. We can see that for regulation with relatively short duration, regulatory uncertainty aggravates the distortion of investment (i.e., $\Gamma_F < \Gamma_U$); for regulation with longer duration, however, *regulatory uncertainty mitigates the distortion* (i.e., $\Gamma_F > \Gamma_U$). This counterintuitive result can be understood as follows. As discussed earlier, regulation distorts investment when the demand shocks reach the investment threshold before the regulation is lifted (i.e., $T_{NN} < T_D$ and $T_{NN} < \tau_D$). When the duration of regulation is short, there is little chance that the demand shocks reach the investment threshold under regulation. In this case, uncertainty in the duration of regulation increases the likelihood of demand shocks hitting the threshold before the regulation is lifted (i.e., $\Gamma_F < \Gamma_U$). When the duration of regulation is significantly long, however, it is highly probable that the demand shocks reach the investment threshold before deregulation. In this case, uncertainty in the duration of regulation rather reduces the likelihood of demand shocks hitting the threshold under regulation (i.e., $\Gamma_F > \Gamma_U$).

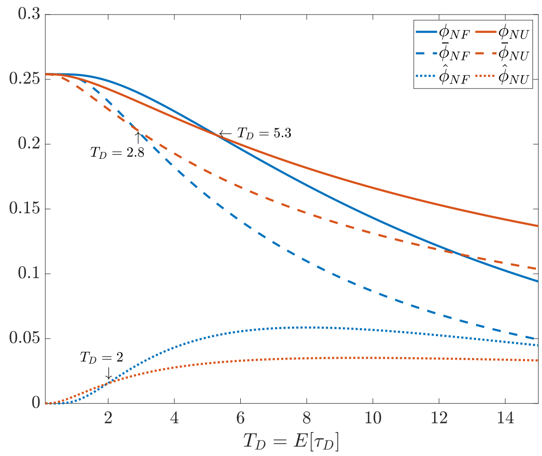
The state prices of investment in Figure 3c verify the aforementioned argument. It shows that for regulation with relatively short duration, uncertainty in regulation delays investment (i.e., $\phi_{NF} > \phi_{NU}$), but for regulation with longer duration, it rather *accelerates investment* (i.e., $\phi_{NF} < \phi_{NU}$). The flip side of this result is described in Figure 3d. For regulation with relatively short duration, uncertainty in its duration aggravates the distortion in the investment induced by regulation (i.e., $\tilde{\phi}_{NF} < \tilde{\phi}_{NU}$); for regulation with longer duration, its uncertainty *mitigates the distortion* (i.e., $\tilde{\phi}_{NF} > \tilde{\phi}_{NU}$). Clearly, firm value decreases as its investment decision is more likely to be distorted. Thus, uncertainty in regulation harms firm value when the expected duration of regulation is relatively short (i.e., $V^R_{NU} < V^R_{NF}$), but it rather *enhances the firm value* (i.e., $V^R_{NU} > V^R_{NF}$) when the regulation is expected to last longer, as described in Figure 3e.



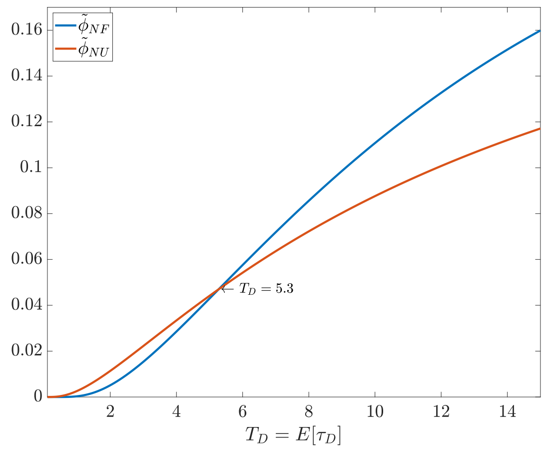
(a) Investment threshold



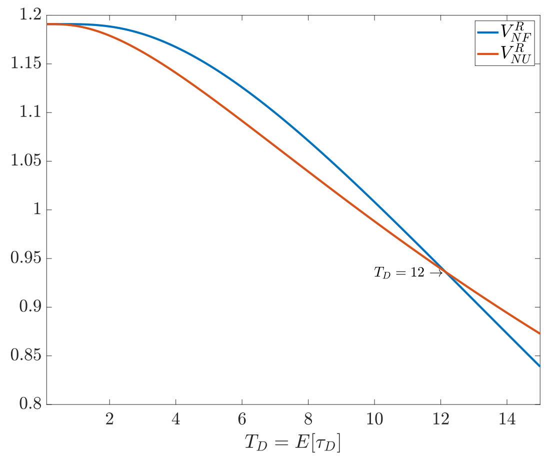
(b) Probability of investment distortion induced by regulation



(c) State prices of investment



(d) Investment distortion measured by the state prices of investment



(e) Initial firm values under regulation

Figure 3: Comparative statics with respect to the duration of regulation in the absence of time-to-build

Note that Γ_F exceeds Γ_U at $T_D = 2.7$ (Figure 3b) but $\tilde{\phi}_{NF}$ starts to dominate $\tilde{\phi}_{NU}$ at $T_D = 5.3$ (Figure 3d); V^R_{NU} becomes greater than V^R_{NF} at $T_D = 12$ (Figure 3e). This discrepancy results from the difference in the magnitude of investment distortion between the investment triggered by deregulation and that not triggered by deregulation. Provided that investment is distorted, the magnitude of the distortion is greater in the latter case than in the former case. That is, the investment in the latter case occurs later than that in the former case (Figures 2a and 2b). Figure 3b shows that $\hat{\Gamma}_F > \hat{\Gamma}_U$ holds for $T_D > 2.5$ but $\bar{\Gamma}_F > \bar{\Gamma}_U$ for $T_D > 2.9$, amplifying the impact of uncertainty of regulation on the investment distortion. Thus, $\tilde{\phi}_{NF}$ exceeds $\tilde{\phi}_{NU}$ at $T_D = 5.3$, long after Γ_F starts to dominate Γ_U. This effect is more amplified for firm value, resulting in $V_{NF} < V_{NU}$ for $T_D > 12$.

6 With time-to-build and regulation

In this section, we assume that both time-to-build and regulation exist. That is, the technology underlying the project requires time-to-build for development and commercialization and conflicts with existing laws and institutions (e.g., robotaxis, drone delivery, airtaxis,[22] private spaceflight,[23] biotechnology-related businesses[24]). In Section 6.1, we assume both time-to-build and deregulation timing are known with certainty; Section 6.2 assumes both are uncertain.[25] Section 6.3 compares the results of each case.

6.1 Fixed time-to-build and deregulation timing

Suppose that both the length of time-to-build and the duration of regulation are constant, denoted by T_B and T_D, respectively. Unlike the case without time-to-build in Section 5, it can be optimal for the firm to invest under regulation when the investment involves time-to-build. Intuitively, the firm can choose to invest under regulation so that the regulation will be lifted by the time the investment project is finished. This argument can be formalized as follows:

Lemma 2 *When time-to-build and the duration of regulation are constants T_B and T_D, respectively, the firm makes a decision of whether to invest immediately or not at $\bar{T}_D := \max(T_D - T_B, 0)$. That is, it can be optimal for the firm to invest under regulation.*

PROOF See Appendix A.12.

Note that if $T_D \le T_B$, the decision to make an immediate investment can only occur at the initial timing (i.e., $t = 0$). In other words, the investment is always triggered by the demand shocks reaching the optimal investment threshold (unless the initial demand shocks are sufficiently high that the investment is triggered at time 0). That is, the firm makes the investment decision *as if there were no regulation.* From the policy perspective, this implies that fixed-duration regulation lasting less than the length of fixed time-to-build *does not distort the firm's investment decision and does not harm the firm value*, which will be proven and discussed in detail in Section 6.3.

If the firm's option to invest is not exercised at $\bar{T}_D$, then the firm value thereafter (i.e., $t \ge \bar{T}_D$) can be described as follows:

$$V_{FF}^D(X) = \max_{T_{FF} \ge 0} \mathbb{E}\Big[\int_{\hat{T}_{FF}}^{\infty} e^{-rt} X_t \mathrm{d}t - e^{-rT_{FF}} C \Big| X_0 = X\Big]. \tag{40}$$

where $T_{FF} := \inf\{t \ge \bar{T}_D | X_t \ge X_{FF}\}$ and $\hat{T}_{FF} := T_{FF} + T_B$ denote the timing of investment and that of project's completion, respectively; X_{FF} denotes the investment threshold with fixed time-to-build and deregulation timing. Note that (40) is equivalent to $V_{FN}(X)$ in (2), provided that the firm has not invested until $\bar{T}_D$, and thus, the firm value $V_{FF}^D(X)$ and the investment threshold X_{FF} are identical to (3) and (4), respectively. Note that although we use the superscript D in (40) for consistency, (40) can be the firm value under regulation (i.e., $t \in [\bar{T}_D, T_D)$).

[22] In the introduction, we noted that the technology behind robotaxis, drone delivery, and airtaxis is nearly in place and only regulation holds back the introduction of the technology to the market. However, it took years for the firms to develop such state-of-the-art technologies, and from the firm's perspective, both time-to-build and regulation existed before making the investment.

[23] In the U.S., the Commercial Space Launch Act of 1984 was passed to facilitate the private commercialization of space and space technology, but private spaceflight remained effectively illegal until 2004. The Commercial Space Launch Amendments Act of 2004 provided a legal framework for commercial human spaceflight. On September 28 2008, Falcon 1 launched by SpaceX, which was founded in 2002, became the first privately-developed fully liquid-fueled rocket to reach orbit around the Earth.

[24] For instance, Neuralink, co-founded by Elon Musk in 2016, has been developing a device inserted into the brain for the purpose of decoding brain activity and linking it to computers. In May 2023, they received approval from the U.S. Food and Drug Administration (FDA) for their first-in-human clinical trial. Clearly, it will take a few decades to fully develop the technology and commercialize it with the clearance of regulation.

[25] In the Online Appendix, we investigate the case in which time-to-build is known with certainty but regulation is uncertain and vice versa.

As addressed in Lemma 2, the firm decides whether to make an immediate investment at $\bar{T}_D$. If the demand by that time exceeds the investment threshold (i.e., $X_{\bar{T}_D} \geq X_{FF}$), the firm's option to invest, which can be read as a European option with maturity $\bar{T}_D$, is exercised immediately. Otherwise (i.e., $X_{\bar{T}_D} < X_{FF}$), the firm's option is an American option that can be exercised at any time thereafter. Thus, the firm value at the initial timing under regulation can be described as follows:[26]

$$\begin{aligned} V_{FF}^R(X) &= \mathbb{E}\big[e^{-r\bar{T}_D} V_{FF}^D(X_{\bar{T}_D})\big|X_0 = X\big] \\ &= \mathbb{E}\Big[\mathbf{1}_{\{X_{\bar{T}_D} \geq X_{FF}\}} e^{-r\bar{T}_D}\Big(\frac{X_{\bar{T}_D} e^{-(r-\mu)T_B}}{r-\mu} - C\Big) \\ &\quad + \mathbf{1}_{\{X_{\bar{T}_D} < X_{FF}\}} e^{-r\bar{T}_D}\Big(\frac{X_{FF} e^{-(r-\mu)T_B}}{r-\mu} - C\Big)\Big(\frac{X_{\bar{T}_D}}{X_{FF}}\Big)^{\beta}\Big|X_0 = X\Big]. \end{aligned} \tag{41}$$

Following similar arguments to those for Proposition 5, the firm value in (41) can be evaluated as follows:[27]

Proposition 8 (Fixed time-to-build and deregulation timing) *Given demand shock X, the firm value at the initial timing with fixed time-to-build T_B and deregulation timing T_D is*

$$V_{FF}^R(X) = \frac{X e^{-(r-\mu)(\bar{T}_D+T_B)}}{r-\mu} N(d_4) - e^{-r\bar{T}_D} C N(d_5) + \Big(\frac{X_{FF} e^{-(r-\mu)T_B}}{r-\mu} - C\Big)\Big(\frac{X}{X_{FF}}\Big)^{\beta} N(d_6), \tag{42}$$

where

$$d_4 := \frac{\ln \frac{X}{X_{FF}} + (\mu + \frac{\sigma^2}{2})\bar{T}_D}{\sigma\sqrt{\bar{T}_D}}, \tag{43}$$

$$d_5 := d_4 - \sigma\sqrt{\bar{T}_D}, \tag{44}$$

$$d_6 := -d_5 - \beta\sigma\sqrt{\bar{T}_D}. \tag{45}$$

PROOF See Appendix A.13.

The first and second rows of (42) correspond to the first and second rows of (41) (i.e., the cases of $X_{\bar{T}_D} \geq X_{FF}$ and $X_{\bar{T}_D} < X_{FF}$), respectively. Note that for $X < X_{FF}$, $\lim_{\bar{T}_D \to 0} d_4 = -\infty$, $\lim_{\bar{T}_D \to 0} d_5 = -\infty$, and $\lim_{\bar{T}_D \to 0} d_6 = \infty$ hold, and thus, $V_{FF}^R(X)$ converges to $V_{FF}^D(X)$, which is identical to $V_{FN}(X)$ in the upper case of (3). For $X \geq X_{FF}$, $\lim_{\bar{T}_D \to 0} d_4 = \infty$, $\lim_{\bar{T}_D \to 0} d_5 = \infty$, and $\lim_{\bar{T}_D \to 0} d_6 = -\infty$, which also amounts to the convergence of $V_{FF}^R(X)$ to $V_{FF}^D(X)$, which is identical to $V_{FN}(X)$ in the lower case of (3). It is straightforward to see that $V_{FF}^R(X)$ converges to $V_{NF}^R(X)$ in (17) as $T_B \to 0$.

6.2 Uncertain time-to-build and deregulation timing

Suppose that both the time-to-build and timing of deregulation, denoted by τ_B and τ_D, respectively, follow the exponential distribution with parameters λ_B and λ_D. As in the case with fixed time-to-build and deregulation timing discussed in Section 6.1, it can be optimal for the firm to invest under regulation. Unlike the case in Section 6.1, however, there are two investment thresholds: one for the investment under deregulation and the other for that under regulation, which we discuss in detail below.

By backward induction, suppose that the regulation has already been lifted (i.e., $t \geq \tau_D$) and the firm has not invested under regulation. In this case, the firm value after deregulation can be described as follows:

$$V_{UU}^D(X) = \max_{T_{UU}^D \geq 0} \mathbb{E}\Big[\int_{\hat{T}_{UU}^D}^{\infty} e^{-rt} X_t \mathrm{d}t - e^{-rT_{UU}^D} C\Big|X_0 = X\Big], \tag{46}$$

where $T_{UU}^D := \inf\{t \geq \tau_D | X_t \geq X_{UU}^D\}$ and X_{UU}^D denote the investment timing and investment threshold after deregulation, respectively, while $\hat{T}_{UU}^D := T_{UU}^D + \tau_B$ denotes the timing of the project's completion, which is uncertain, unlike $\hat{T}_{FF}$. Note that (46) is equivalent to $V_{UN}(X)$ in (6). Thus, the firm value $V_{UU}^D(X)$ and the investment threshold X_{UU}^D are identical to (7) and (8), respectively.

[26] For $t \in (0, \bar{T}_D)$, the firm value is $\mathbb{E}[e^{-r(\bar{T}_D - t)} V_{FF}^D(X_{\bar{T}_D}) | X_t = X]$.

[27] For $t \in (0, \bar{T}_D)$, the firm value can be evaluated as (42) through (45) with $\bar{T}_D - t$ instead of $\bar{T}_D$.

Now, let us suppose that the regulation is still in place (i.e., $t < \tau_D$). Because both time-to-build and the timing of deregulation are uncertain, it can be optimal for the firm to invest under regulation with the expectation that the regulation will be lifted by the time the investment project is finished. Although the firm invests under regulation with such expectation, it is possible that the regulation has not been lifted by the time the project is finished, obstructing the product's market entry. Thus, the firm value under regulation can be described as follows:[28]

$$
\begin{aligned}
V_{UU}^R(X) &= \max_{T_{UU}^R \geq 0} \mathbb{E}\Big[\mathbf{1}_{\{T_{UU}^R < \tau_D\}}\Big\{\int_{\hat{T}_{UU}^R \vee \tau_D}^{\infty} e^{-rt} X_t \mathrm{d}t - e^{-rT_{UU}^R} C\Big\} + \mathbf{1}_{\{\tau_D \leq T_{UU}^R\}} e^{-r\tau_D} V_{UU}^D(X_{\tau_D}) \Big| X_0 = X\Big] \\
&= \max_{T_{UU}^R \geq 0} \mathbb{E}\Big[\mathbf{1}_{\{T_{UU}^R < \tau_D \leq \hat{T}_{UU}^R\}}\Big\{\int_{\hat{T}_{UU}^R}^{\infty} e^{-rt} X_t \mathrm{d}t - e^{-rT_{UU}^R} C\Big\} + \mathbf{1}_{\{\hat{T}_{UU}^R < \tau_D\}}\Big\{\int_{\tau_D}^{\infty} e^{-rt} X_t \mathrm{d}t - e^{-rT_{UU}^R} C\Big\} \\
&\qquad + \mathbf{1}_{\{\tau_D \leq T_{UU}^R\}} V_{UU}^D(X_{\tau_D}) \Big| X_0 = X\Big]
\end{aligned} \tag{47}
$$

where $T_{UU}^R := \inf\{t \in [0, \tau_D) | X_t \geq X_{UU}^R\}$ denotes the investment timing with the investment threshold under regulation X_{UU}^R, while $\hat{T}_{UU}^R := T_{UU}^R + \tau_B$ denotes the timing of project's completion, which is uncertain. The first and second terms on the right-hand side of (47) represent the case of investment under regulation; the former and the latter correspond to the cases in which regulation is lifted before and after the project's completion (i.e., $T_{UU}^R < \tau_D \leq \hat{T}_{UU}^R$ and $\hat{T}_{UU}^R < \tau_D$), respectively. The last term of (47) represents the case in which the regulation is lifted before the market demand reaches the investment threshold under regulation, such that the firm invests after deregulation as described in (46) (i.e., $\tau_D \leq T_{UU}^R$).

Following similar arguments as in Proposition 6, the firm value under regulation in (47) can be evaluated as follows:

Proposition 9 (Uncertain time-to-build and deregulation timing) *Given demand shock X, the firm value under regulation with uncertain time-to-build τ_B and deregulation timing τ_D is*

$$
V_{UU}^R(X) = \begin{cases}
\Big[\frac{\lambda_B \lambda_D X_{UU}^R}{(r+\lambda_B-\mu)(r+\lambda_D-\mu)(r+\lambda_B+\lambda_D-\mu)} - \frac{rC}{r+\lambda_D}\Big]\Big(\frac{X}{X_{UU}^R}\Big)^{\beta_D} \\
\quad + \Big[\frac{\lambda_B X_{UU}^D}{(r-\mu)(r+\lambda_B-\mu)} - C\Big]\Big(\frac{X}{X_{UU}^D}\Big)^{\beta} - \Big[\frac{\lambda_B X_{UU}^D}{(r+\lambda_B-\mu)(r+\lambda_D-\mu)} - \frac{rC}{r+\lambda_D}\Big]\Big(\frac{X}{X_{UU}^D}\Big)^{\beta_D} \\
\quad + \frac{1}{\beta_D-\gamma_D}\Big[(\beta_D-\beta)\Big\{\frac{\lambda_B X_{UU}^D}{(r-\mu)(r+\lambda_B-\mu)} - C\Big\} - \Big\{\frac{(\beta_D-1)\lambda_B\lambda_D X_{UU}^D}{(r-\mu)(r+\lambda_B-\mu)(r+\lambda_D-\mu)} - \frac{\beta_D \lambda_D C}{r+\lambda_D}\Big\}\Big] \\
\quad \times \Big\{\Big(\frac{X}{X_{UU}^D}\Big)^{\beta_D} - \Big(\frac{X}{X_{UU}^R}\Big)^{\beta_D}\Big(\frac{X_{UU}^R}{X_{UU}^D}\Big)^{\gamma_D}\Big\}, & \textit{if } X < X_{UU}^D, \\
\frac{\lambda_B \lambda_D X}{(r-\mu)(r+\lambda_B-\mu)(r+\lambda_D-\mu)} - \frac{\lambda_D C}{r+\lambda_D} + \Big[\frac{\lambda_B \lambda_D X_{UU}^R}{(r+\lambda_B-\mu)(r+\lambda_D-\mu)(r+\lambda_B+\lambda_D-\mu)} - \frac{rC}{r+\lambda_D}\Big]\Big(\frac{X}{X_{UU}^R}\Big)^{\beta_D} \\
\quad + \frac{1}{\beta_D-\gamma_D}\Big[(\beta_D-\beta)\Big\{\frac{\lambda_B X_{UU}^D}{(r-\mu)(r+\lambda_B-\mu)} - C\Big\} - \Big\{\frac{(\beta_D-1)\lambda_B\lambda_D X_{UU}^D}{(r-\mu)(r+\lambda_B-\mu)(r+\lambda_D-\mu)} - \frac{\beta_D \lambda_D C}{r+\lambda_D}\Big\}\Big] \\
\quad \times \Big\{\Big(\frac{X}{X_{UU}^D}\Big)^{\gamma_D} - \Big(\frac{X}{X_{UU}^R}\Big)^{\beta_D}\Big(\frac{X_{UU}^R}{X_{UU}^D}\Big)^{\gamma_D}\Big\}, & \textit{if } X_{UU}^D \leq X < X_{UU}^R, \\
\frac{\lambda_B \lambda_D X}{(r-\mu)(r+\lambda_B+\lambda_D-\mu)}\Big(\frac{1}{r+\lambda_B-\mu} + \frac{1}{r+\lambda_D-\mu}\Big) - C, & \textit{if } X \geq X_{UU}^R,
\end{cases} \tag{48}
$$

where the optimal investment thresholds under regulation, X_{UU}^R, is implicitly derived from

$$
\begin{aligned}
&\frac{(\beta_D-1)\lambda_B\lambda_D X_{UU}^R}{(r+\lambda_B-\mu)(r+\lambda_D-\mu)(r+\lambda_B+\lambda_D-\mu)} - \frac{r\lambda_B C}{r+\lambda_D} \\
&= C\Big(\frac{\beta_D-\beta}{\beta-1} - \frac{\beta(\beta_D-1)}{(\beta-1)(r+\lambda_D-\mu)} + \frac{\beta_D\lambda_D}{r+\lambda_D}\Big)\Big(\frac{X_{UU}^R}{X_{UU}^D}\Big)^{\gamma_D}.
\end{aligned} \tag{49}
$$

Proof See Appendix A.14.

The first row of the upper case in (48) corresponds to the first term in the first row of (47) (i.e., $T_{UU}^R < \tau_D$), while the sum of the second, third, and fourth rows of the upper case in (48) corresponds to the second term in the first row of (47) (i.e., $\tau_D \leq T_{UU}^R$). The first term of (47) can be evaluated relatively simply because it is associated with the demand shock reaching the threshold X_{UU}^R for the first time *before* exponential time τ_D. The evaluation of the second term of (47) is complex because it is associated with the demand shock hitting the threshold X_{UU}^D *after* exponential time τ_D. It can be the first time the demand shock hits X_{UU}^D and happens to

[28]Given demand shock X, the firm value under regulation is the same as (47) for all $t \in [0, \tau_D)$ due to memoryless property of exponential distribution.

be after deregulation at the same time, which corresponds the second row of (48); it can be not the first time hitting X_{UU}^D but after repeatedly going in and out of the region (X_{UU}^D, X_{UU}^R) without hitting X_{UU}^R and happens to be below X_{UU}^D when the regulation is lifted, which corresponds to the third and fourth rows of (48).[29]

As $\lambda_D \to \infty$, $\beta_D \to \infty$ and $\gamma_D \to -\infty$; thus, $V_{UU}^R(X)$ converges to $V_{UN}(X)$ in (7). Meanwhile, X_{UU}^D converges to X_{NU} as $\lambda_B \to \infty$. Although X_{UU}^R is not explicitly derived, we can conjecture from Lemma 1 that $X_{UU}^R \to \infty$ as $\lambda_B \to \infty$. With these, it is straightforward to see that $V_{UU}^R(X)$ converges to $V_{NU}^R(X)$ in (23) as $\lambda_B \to \infty$.

6.3 Model comparison and discussion

When both time-to-build and deregulation timing are fixed, there is a single investment threshold and the earliest investment is the one triggered at $\bar{T}_D$; regardless of the level of demand, the firm never invests before $\bar{T}_D$. By contrast, when both time-to-build and deregulation timing are uncertain, there are two investment thresholds, one of which can only be derived by numerical calculation. For this reason, we cannot compare the likelihood of investment between two cases analytically in the fashion of Propositions 3 and 7. However, we can derive the state prices of investment in the presence of time-to-build and regulation, following the same arguments as in Propositions 8 and 9, and compare how uncertainty in time-to-build and regulation affects the firm's investment decision:

Corollary 4 (State prices of investment with time-to-build and regulation) *Given demand shock X, the state price of investment with fixed time-to-build and deregulation timing is*

$$\phi_{FF}(X) := \mathbb{E}\big[e^{-rT_{FF}}\big|X_0 = X\big] = \hat{\phi}_{FF}(X) + \bar{\phi}_{FF}(X), \tag{50}$$

where

$$\hat{\phi}_{FF}(X) := \mathbb{E}\big[\mathbf{1}_{\{X_{\bar{T}_D} \geq X_{FF}\}} e^{-r\bar{T}_D}\big|X_0 = X\big] = e^{-r\bar{T}_D} N(d_5), \tag{51}$$

$$\bar{\phi}_{FF}(X) := \mathbb{E}\big[\mathbf{1}_{\{X_{\bar{T}_D} < X_{FF}\}} e^{-rT_{FF}}\big|X_0 = X\big] = \Big(\frac{X}{X_{FF}}\Big)^{\beta} N(d_6), \tag{52}$$

denote the state prices of investment triggered at $\bar{T}_D$, which is always under regulation, and at the first-hitting time of the investment threshold after $\bar{T}_D$, which can be either under regulation or deregulation, respectively.

Similarly, the state price of investment with uncertain time-to-build and deregulation timing is

$$\phi_{UU}(X) := \mathbb{E}\big[\mathbf{1}_{\{\tau_D \leq T_{UU}^D\}} e^{-rT_{UU}^D} + \mathbf{1}_{\{T_{UU}^R < \tau_D\}} e^{-rT_{UU}^R}\big|X_0 = X\big] = \hat{\phi}_{UU}^D(X) + \bar{\phi}_{UU}^D(X) + \bar{\phi}_{UU}^R(X), \tag{53}$$

where

$$\begin{aligned}\hat{\phi}_{UU}^D(X) :=& \mathbb{E}\big[\mathbf{1}_{\{\tau_D = T_{UU}^D\}} e^{-r\tau_D}\big|X_0 = X\big] \\ =& \frac{\lambda_D}{r+\lambda_D}\Big[\Big(\frac{X}{X_{UU}^D}\Big)^{\beta_D} - \Big(\frac{X}{X_{UU}^R}\Big)^{\beta_D} - \frac{\beta_D}{\beta_D - \gamma_D}\Big\{\Big(\frac{X}{X_{UU}^D}\Big)^{\beta_D} - \Big(\frac{X}{X_{UU}^R}\Big)^{\beta_D}\Big(\frac{X_{UU}^R}{X_{UU}^D}\Big)^{\gamma_D}\Big\}\Big], \quad (54)\\ \bar{\phi}_{UU}^D(X) :=& \mathbb{E}\big[\mathbf{1}_{\{\tau_D < T_{UU}^D\}} e^{-rT_{UU}^D}\big|X_0 = X\big] \\ =& \Big(\frac{X}{X_{UU}^D}\Big)^{\beta} - \frac{\beta - \gamma_D}{\beta_D - \gamma_D}\Big(\frac{X}{X_{UU}^D}\Big)^{\beta_D} - \frac{\beta_D - \beta}{\beta_D - \gamma_D}\Big(\frac{X}{X_{UU}^R}\Big)^{\beta_D}\Big(\frac{X_{UU}^R}{X_{UU}^D}\Big)^{\gamma_D}, \quad (55)\\ \bar{\phi}_{UU}^R(X) :=& \mathbb{E}\big[\mathbf{1}_{\{T_{UU}^R < \tau_D\}} e^{-rT_{UU}^R}\big|X_0 = X\big] = \Big(\frac{X}{X_{UU}^R}\Big)^{\beta_D}, \quad (56)\end{aligned}$$

denote the state prices of investment triggered by deregulation, hitting the investment threshold under deregulation, and hitting the investment threshold under regulation, respectively.

Proof See Appendix A.15.

As in Section 5.3, we can measure the distortion in investment induced by regulation with fixed and uncertain duration as $\tilde{\phi}_{FF} := \phi_{FN} - \phi_{FF}$ and $\tilde{\phi}_{UU} := \phi_{UN} - \phi_{UU}$, respectively. From the argument in Lemma 2, we can obtain the following result:

[29] It is natural that the firm has much more incentive to invest after deregulation than under regulation, which can be represented as $X_{UU}^D < X_{UU}^R$.

Proposition 10 (Harmless regulation) *Regulation with fixed duration no longer than fixed time-to-build does not induce any distortion in investment decision, nor does it harm any firm value:*

$$\tilde{\phi}_{FF}(X) = 0 \quad \textit{and} \quad V^R_{FF}(X) = V^R_{NF}(X) \qquad \textit{if } T_D \leq T_B. \tag{57}$$

Proof See Appendix A.16.

In the real world, strict regulation that prohibits the commercialization of products or services based on radical technologies can be necessary to gain time for legal arrangements, even at the cost of delaying the introduction of new technologies to the market. For instance, robotaxis, drone delivery, and aixtaxis will cause severe social disruption unless traffic laws and insurance systems are amended accordingly. The implantation of computers into the human brain is ethically controversial and can cause social problems without sufficient discussion and an appropriate legal system. Note that *we have shown the existence of regulation that does not harm social welfare, even without introducing the explicit benefits of preventing social disruption* due to the lack of relevant legislation.

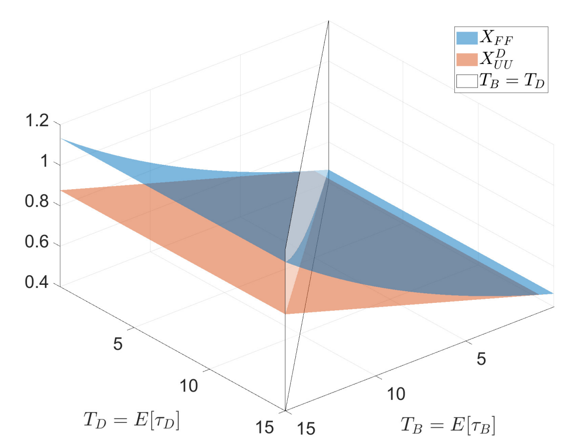
Figure 4 presents the comparative statics regarding the length of time-to-build (i.e., $T_B = 1/\lambda_B$) and the duration of regulation (i.e., $T_D = 1/\lambda_D$) based on the parameters in Table 1 to compare the models from Sections 6.1 and 6.2.

As noted earlier, there is a single investment threshold if both time-to-build and regulation are known with certainty (i.e., X_{FF}), whereas there are two thresholds if they are uncertain (i.e., X^D_{UU} and X^R_{UU}). Figure 4a compares X_{FF} and X^D_{UU}, which trigger investment after regulation becomes irrelevant to the investment decision (i.e., $t \geq \bar{T}_D$ and $t \geq \tau_D$), and we can see that $X^D_{UU} < X_{FF}$ always holds. This is obvious considering the result in Proposition 3 with $X_{FF} = X_{FN}$ and $X^D_{UU} = X_{UN}$. Figure 4b compares the two investment thresholds in the presence of uncertain time-to-build and regulation (i.e., X^D_{UU} and X^R_{UU}). We can see that the gap between them widens as expected time-to-build shortens and the expected duration of regulation lengthens. This is a natural result considering that the investment project made under regulation is more likely to be finished before deregulation, which is the last outcome the firm would want.

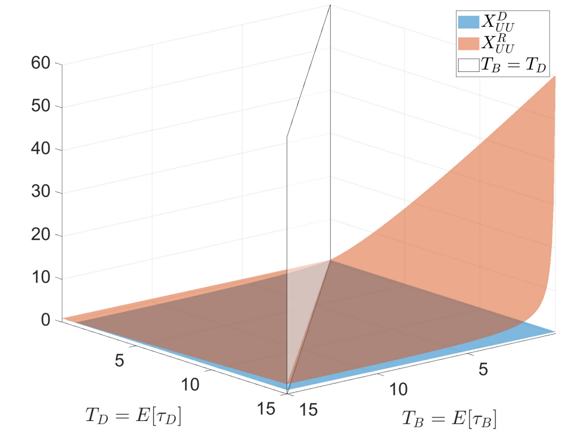
Investment under regulation is in line with Gulen and Ion (2016). They noted that while firms delay investments in the face of policy uncertainty, they may have no choice but to invest eventually, either because the projects cannot be delayed indefinitely or because the cash flows lost by the delay become too large to justify further delays. This claim is supported by their empirical analysis; average investments decrease due to an increase of policy uncertainty, but they eventually recover to the level of investment with below-average policy uncertainty after seven or more quarter of high policy uncertainty.

The decomposition of the state price of investment with fixed time-to-build and deregulation timing described in Figure 4c verifies the argument in Lemma 2. For $T_D \leq T_B$, the investment is never triggered at $\bar{T}_D$ (i.e., $\hat{\phi}_{FF} = 0$); it is always made at the first-hitting time of the investment threshold thereafter. For $T_D > T_B$, however, investment can be made immediately at $\bar{T}_D$, which is always under regulation (i.e., $\hat{\phi}_{FF} > 0$). Note that the total likelihood of investment (i.e., ϕ_{FF}) does not strictly decrease with the size of time-to-build when the duration of regulation is sufficiently long. That is, it is possible that *the likelihood of investment increases with the size of investment lags.* This is because when the timing of deregulation is known with certainty, the firm can choose to invest earlier in a project that takes longer to complete, taking the lead time into account. This argument, however, does not hold under uncertain time-to-build and regulation.

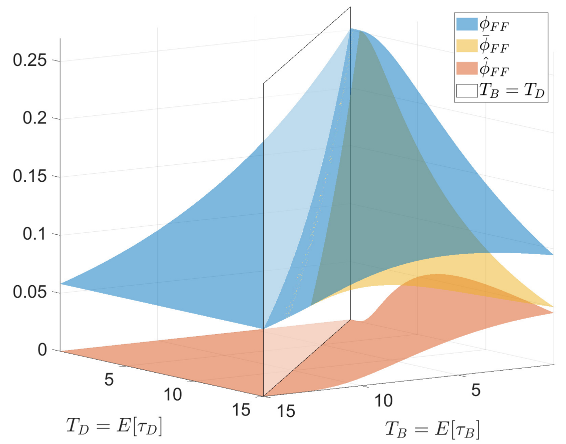
The decomposition of the state price of investment in the presence of uncertain time-to-build and regulation described in Figure 4d is associated with the investment thresholds in Figure 4b. The total likelihood of investment in this case (i.e., ϕ_{UU}) strictly decreases with the size of time-to-build, regardless of the duration of regulation. It is natural that the likelihood of investment triggered by deregulation (i.e., $\hat{\phi}^D_{UU}$) is higher when regulation is expected to last longer but expected time-to-build is relatively short. Additionally, the likelihood of investment triggered under regulation (i.e., $\bar{\phi}^R_{UU}$) is higher when both time-to-build and regulation are expected to last longer. Note that although the gap between X^D_{UU} and X^R_{UU} increases with $\mathbb{E}[\tau_D]$ (Figure 4b), the state price of investment under regulation (i.e., $\bar{\phi}^R_{UU}$) remains high when both $\mathbb{E}[\tau_B]$ and $\mathbb{E}[\tau_D]$ are high (Figure 4d). This is because the duration of regulation is long enough that demand shocks are likely to reach the investment threshold under regulation (i.e., X^R_{UU}) before the regulation is lifted, though the threshold increases with $\mathbb{E}[\tau_D]$.



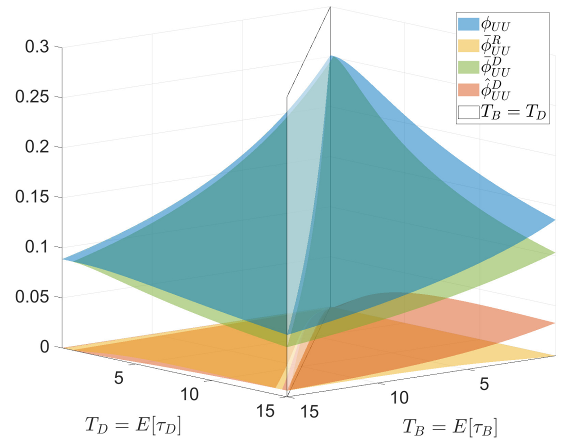
(a) Investment thresholds irrelevant to regulation



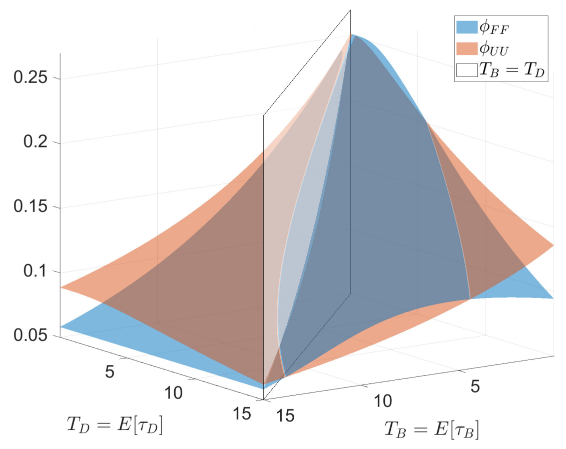
(b) Investment thresholds with uncertain time-to-build and regulation



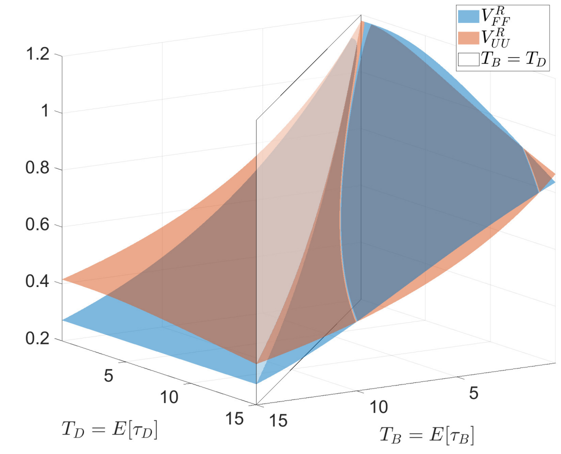
(c) Decomposition of state price of investment with fixed time-to-build and regulation



(d) Decomposition of state price of investment with uncertain time-to-build and regulation



(e) State prices of investment



(f) Initial firm values under regulation

Figure 4: Comparative statics with respect to the size of time-to-build and the duration of regulation

Figure 4e compares the state price of investment with fixed time-to-build and regulation to that with uncertain time-to-build and regulation (i.e., ϕ_{FF} and ϕ_{UU}). We can see that for $T_D \leq T_B$, *uncertainty in time-to-build and regulation accelerates investment* (i.e., $\phi_{UU} > \phi_{FF}$ for most cases). This is mainly because the presence of regulation does not affect the investment decision with fixed time-to-build and deregulation timing for $T_D \leq T_B$

(Lemma 2), and $X^D_{UU} < X_{FF}$ always holds for the investment thresholds irrelevant to the regulation (Figure 4a). In addition, if both time-to-build and regulation are uncertain, investment can occur under regulation (i.e., $\bar{\phi}^R_{UU} > 0$ in Figure 4d).

For $T_D > T_B$, however, the opposite holds; uncertainty in time-to-build and regulation delays investment (i.e., $\phi_{UU} < \phi_{FF}$ for most cases). This is mainly because as T_B decreases and T_D increases, the investment with fixed time-to-build and regulation is more likely to be triggered at $\bar{T}_D$, which is the earliest investment timing (Lemma 2). Recall that X_{FF} strictly increases with T_B (Figure 4a). When T_B is insignificant and T_D is substantial, however, $\phi_{FF} < \phi_{UU}$ holds again. Intuitively, this is because when time-to-build is insignificant and lengthy regulation is certain, it is highly probable that regulation delays investment; if there is uncertainty in the duration of regulation, then the regulation may be lifted earlier than expected and the firm invests instantly (i.e., $\hat{\phi}^D_{UU}$ in Figure 4d).

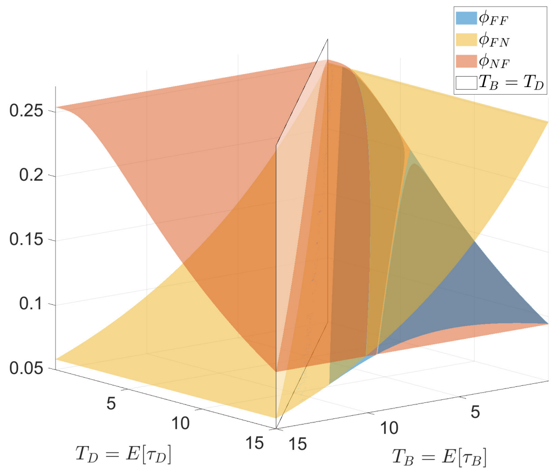
The same argument can explain the firm value described in Figure 4f; $V^R_{UU} > V^R_{FF}$ holds for $T_D \le T_B$, but $V^R_{UU} < V^R_{FF}$ for most cases in $T_D > T_B$. We omit the repetitive explanation.

Figure 5 compares the models discussed in Sections 4 to 6. Figure 5a compares the state prices of investment in the absence of uncertainty (Sections 4.1, 5.1 and 6.1). In Figures 1a and 1b, we showed that *uncertainty in time-to-build* always accelerates investment in the absence of regulation (i.e., $X_{UN} < X_{FN}$ and $\phi_{UN} > \phi_{FN}$) but *the presence of time-to-build* always delays investment, whether it is certain or not (i.e., X_{FN} and X_{UN} strictly increase with $T_B = \mathbb{E}[\tau_B]$). By contrast, Figure 5a clearly shows that *the presence of time-to-build can accelerate investment* in the presence of regulation (i.e., $\phi_{FF} > \phi_{NF}$). This counterintuitive result is directly associated with Lemmas 1 and 2. Without time-to-build, the firm never invests under regulation (Lemma 1). However, if the investment project involves fixed time-to-build and the timing of deregulation is known with certainty, the firm can optimally choose to invest under regulation, taking the lead time into account (Lemma 2). In this sense, time-to-build can advance the timing of investment in the presence of regulation.

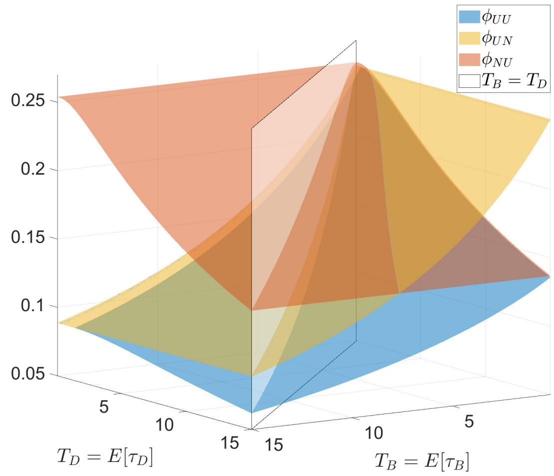
This novel result is in contrast with extant studies on the impact of time-to-build on corporate investment. Majd and Pindyck (1987) assumed a maximum rate at which a firm can invest and showed that time-to-build delays investment. Bar-Ilan and Strange (1996*a*,*b*) showed that uncertainty in output prices or demand can accelerate investment if the project involves time-to-build. However, they assumed that a firm can abandon an ongoing project in accordance with demand, which truncates the downside risk of investment; it was essentially the existence of the abandonment option that yields the positive impact of uncertainty on investment. By contrast, this study concentrates on uncertainty in the timing of revenue generation and shows that not only uncertainty of time-to-build but also its presence can accelerate investment, even without assuming the firm's option to truncate the downside risk.

Figure 5b compares the state prices of investment with uncertainty in time-to-build and regulation (Sections 4.2, 5.2 and 6.2). It shows that when there is uncertainty in both time-to-build and regulation, the presence of time-to-build always delays investment (i.e., $\phi_{UU} < \phi_{NU}$), which is in sharp contrast with the case with fixed time-to-build and regulation (i.e., $\phi_{FF} > \phi_{NF}$). Figures 5a and 5b clarify that *the presence of regulation does not accelerate investment*, whether it is uncertain or not (i.e., $\phi_{FF} < \phi_{FN}$ and $\phi_{UU} < \phi_{UN}$). Note that $\lim_{T_B \to 0} \phi_{FF} = \phi_{NF}$, $\lim_{T_D \to 0} \phi_{FF} = \phi_{FN}$, $\lim_{\mathbb{E}[\tau_B] \to 0} \phi_{UU} = \phi_{NU}$, and $\lim_{\mathbb{E}[\tau_D] \to 0} \phi_{UU} = \phi_{UN}$ hold.

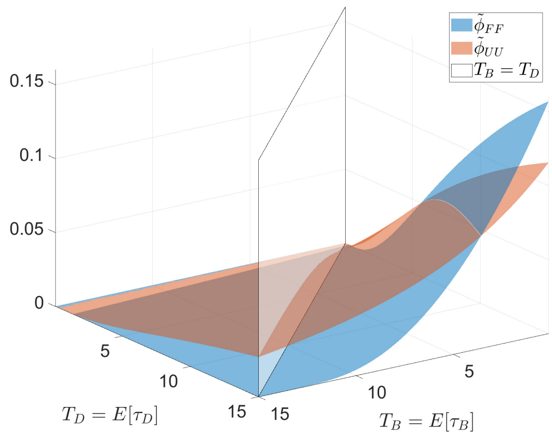
Figures 5c and 5d verify the harmless regulation discussed in Proposition 10; when the fixed duration of regulation is no longer than the fixed time-to-build, the regulation does not distort the investment decision, nor does it harm firm value (i.e., $\tilde{\phi}_{FF} = 0$ and $V^R_{FF} = V_{FN}$ for $T_D \le T_B$). By contrast, Figures 5c and 5e clearly show that *uncertain regulation always distorts the firm's investment decision, and thus, always harms the firm value* (i.e., $\tilde{\phi}_{UU} > 0$ and $V^R_{UU} < V_{UN}$). Figure 5c also shows that in the presence of time-to-build, uncertainty in regulation worsens the distortion in the investment induced by regulation in most cases (i.e., $\tilde{\phi}_{FF} < \tilde{\phi}_{UU}$). When time-to-build is insignificant and the duration of regulation is substantial, however, it is possible that *uncertainty in regulation mitigates the distortionary effect of regulation on investment* (i.e., $\tilde{\phi}_{FF} > \tilde{\phi}_{UU}$). This follows the same argument for $\phi_{FF} > \phi_{UU}$ in Figure 4c, and we omit the repetitive illustration.



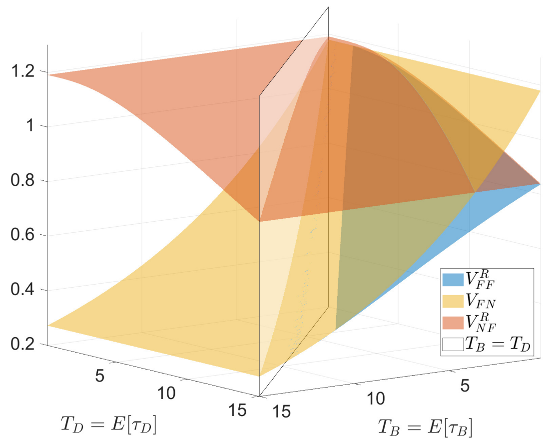
(a) Comparison of state prices of investment with fixed time-to-build and regulation



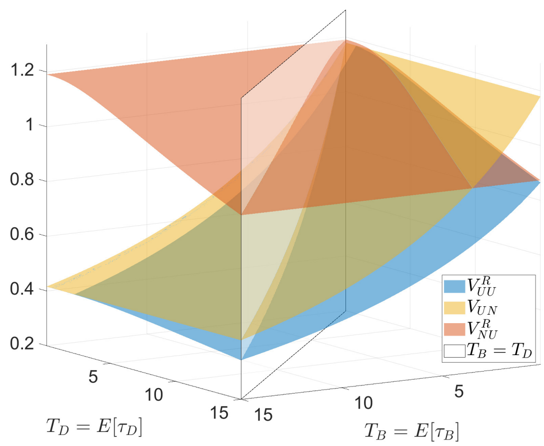
(b) Comparison of state prices of investment with uncertain time-to-build and regulation



(c) Distortion in investment decision by the presence of regulation



(d) Initial firm values with fixed time-to-build and regulation



(e) Initial firm values with uncertain time-to-build and regulation

Figure 5: Comparative statics with respect to the size of time-to-build and the duration of regulation

7 Conclusion

This study investigated the effects of uncertainty in the timing of revenue generation on a firm's investment decision. The firm's revenue generation can be delayed by either the time-to-build of the project or regulation. We showed that in the absence of regulation, uncertainty in time-to-build always accelerates investment and enhances firm value because of the convexity of the discount factor with respect to the timing of revenue generation. We also show that in the absence of time-to-build, uncertainty in regulation can alleviate the distortion of investment decision induced by the regulation. When the duration of regulation is substantial, it is highly probable that the firm would have invested if it had not been for the regulation. In such cases, uncertainty in the duration of regulation can rather reduce the probability of investment delayed by regulation. Furthermore, we proved that in the presence of both time-to-build and regulation, there can exist harmless regulation that does not induce distorted investment decision and does not harm firm value. This result was derived even without introducing the benefits of regulation of preventing social disruption. Lastly, we showed that not only uncertainty of time-to-build but its presence can accelerate investment in the presence of regulation, even without introducing the firm's option to truncate the downside risk of the investment.

Many problems remain to be tackled. For instance, we simplified the modelling by assuming a monopolistic firm, but competition in the market can change the results significantly mainly due to preemptive incentives. A general equilibrium model could enable us to investigate the effects of uncertainty in the timing of revenue generation on industry dynamics. We considered only the firm's investment timing decision to ensure tractability, but future works can also consider the investment size decision by introducing a demand curve, which could enable an analysis of the impacts on consumer surplus and social welfare as well. For simplicity, we considered only an all-equity firm in this study. Incorporating debt financing can extend the analysis to the impacts of uncertainty in the timing of revenue generation on the firm's decisions of capital structure and default. Lastly, we provide only a theoretical framework to investigate the problem, and empirical analysis should be carried our in the future to test the results derived from this model, although it might be challenging to obtain firm-level or project-level data. It is to be hoped that this study will serve as a platform to explore these problems in the future.

A Proofs

A.1 Proof of Proposition 1

It is straightforward that the firm's expected profits at the investment timing for given demand shock X are

$$V(X) = \mathbb{E}\Big[\int_{T_B}^{\infty} e^{-rt}X_t\mathrm{d}t - C\Big|X_0 = X\Big] = \frac{Xe^{-(r-\mu)T_B}}{r-\mu} - C. \tag{58}$$

The value of the option to invest in this project, $F(X)$, should satisfy

$$rF(X) = \mathcal{L}F(X), \tag{59}$$

subject to

$$F(0) = 0, \tag{60}$$

$$F(X_{FN}) = V(X_{FN}), \tag{61}$$

$$\frac{\partial F}{\partial X}\Big|_{X=X_{FN}} = \frac{\partial V}{\partial X}\Big|_{X=X_{FN}}, \tag{62}$$

where the second-order partial differential operator is defined by

$$\mathcal{L}F(X) := \mu X\frac{\partial F(X)}{\partial X} + \frac{1}{2}\sigma^2 X^2\frac{\partial^2 F(X)}{\partial X^2}. \tag{63}$$

A general solution to (59) with the boundary condition (60) is

$$V(X) = AX^{\beta}, \tag{64}$$

where β is given by (5). Substituting (58) and (64) into (61) and (62), we can obtain the results in Proposition 1.

A.2 Proof of Proposition 2

The firm's expected profits at the investment timing for a given demand shock X are

$$\mathbb{E}\Big[\int_{\tau_B}^{\infty} e^{-rt}X_t\mathrm{d}t - C\Big|X_0 = X\Big] = \frac{\lambda_B X}{(r-\mu)(r+\lambda_B-\mu)} - C. \tag{65}$$

Following the same arguments as in (59) through (64), we can obtain the results in Proposition 2.

A.3 Proof of Proposition 3

For $T_B = 1/\lambda_B$, $X_{UN} \leq X_{FN}$ always holds if $f \geq g$ holds for $\lambda_B \in (0,\infty)$, where $f := e^{(r-\mu)/\lambda_B}$ and $g := (r+\lambda_B-\mu)/\lambda_B$. It is straightforward to show that f/g strictly decreases with λ_B and the L'Hôpital rule yields $\lim_{\lambda_B\to\infty} f/g = 1$. Thus, $X_{UN} \leq X_{FN}$ always holds. Substituting (4) and (8) into (3) and (7), respectively, with $X_{UN} \leq X_{FN}$ ensures that $V_{UN} \geq V_{FN}$ always holds.

A.4 Proof of Proposition 4

The proof is based on the well-known equivalence theorem from Rothschild and Stiglitz (1970). By definition, $\tau_B^G = \tau_B^F + \epsilon$ where $\mathbb{E}[\epsilon|\tau_B^F] = 0$. Suppose $v(\cdot)$ is a convex function. By law of iterated expectation and Jensen's inequality, we have

$$\int v(\tau_B^G)\mathrm{d}G(\tau_B^G) = \int \mathbb{E}[v(\tau_B^F+\epsilon|\tau_B^F)]\mathrm{d}F(\tau_B^F) \geq \int v(\mathbb{E}[\tau_B^F+\epsilon|\tau_B^F])\mathrm{d}F(\tau_B^F) = \int v(\tau_B^F)\mathrm{d}F(\tau_B^F). \tag{66}$$

Meanwhile, the firm value at the investment timing with time-to-build τ_B^i for $i \in \{F,G\}$ is

$$\mathbb{E}\Big[\int_{\tau_B^i}^{\infty} e^{-rt}X_t\mathrm{d}t - C\Big|X_0 = X\Big] = \frac{XB(\tau_B^i)}{r-\mu} - C, \tag{67}$$

where $B(\tau) = \mathbb{E}[e^{-(r-\mu)\tau}]$. By the same arguments as Propositions 1 and 2, the firm value having an option to invest in a project with time-to-build τ_B^i for $i \in \{F,G\}$ is

$$V_{UN}^i(X) = \Big[\frac{X_{UN}^i B(\tau_B^i)}{r-\mu} - C\Big]\Big(\frac{X}{X_{UN}^i}\Big)^{\beta}, \tag{68}$$

where the optimal investment threshold is

$$X_{UN}^i = \frac{\beta(r-\mu)C}{(\beta-1)B(\tau_B^i)}. \tag{69}$$

$B(\tau_B^G) \geq B(\tau_B^F)$ by (66), and thus, $X_{UN}^G \leq X_{UN}^F$ and $V_{UN}^G(X) \geq V_{UN}^F(X)$ always hold.

A.5 Proof of Corollary 1

The state price of investment in the absence of regulation with fixed time-to-build T_B should satisfy (59) subject to (60) and $F(X_{FN}) = 1$. Likewise, the state price of investment in the absence of regulation with uncertain time-to-build τ_B should satisfy (59) subject to (60) and $F(X_{UN}) = 1$. The value-matching conditions yield (11) and (12).

A.6 Proof of Lemma 1

Suppose that the firm invests at $T(\leq T_D)$. Though the investment costs occur at T, the firm can raise revenue from T_D, and the firm value before the investment is

$$\max_{T\in[0,T_D]} \mathbb{E}\Big[\int_{T_D}^{\infty} e^{-rt}X_t\mathrm{d}t - e^{-rT}C\Big|X_0 = X\Big] = \mathbb{E}\Big[\int_{T_D}^{\infty} e^{-rt}X_t\mathrm{d}t|X_0 = X\Big] - \max_{T\in[0,T_D]} e^{-rT}C. \tag{70}$$

It is straightforward that the last term on the right-hand side of (70) strictly decreases with T. That is, (70) strictly increases with $T(\leq T_D)$, and thus, the firm never invests before T_D. This argument clearly also holds under a stochastic deregulation timing.

A.7 Proof of Proposition 5

Following the derivation of the well-known Black-Scholes formula for pricing a European-style financial option, the first term on the right-hand side of (16) can be evaluated as follows:

$$
\begin{aligned}
&\mathbb{E}\Big[\mathbf{1}_{\{X_{T_D}\geq X_{NF}\}}e^{-rT_D}\Big(\frac{X_{T_D}}{r-\mu}-C\Big)\Big|X_0=X\Big]\\
&=\frac{e^{-rT_D}}{r-\mu}\mathbb{E}\Big[\mathbf{1}_{\{X_{T_D}\geq X_{NF}\}}Xe^{(\mu-\frac{\sigma^2}{2})T_D+\sigma W_{T_D}}\Big]-e^{-rT_D}C\mathbb{P}\Big(Xe^{(\mu-\frac{\sigma^2}{2})T_D+\sigma W_{T_D}}\geq X_{NF}\Big)\\
&=\frac{Xe^{-(r-\mu)T_D}}{r-\mu}e^{-\frac{\sigma^2T_D}{2}}\mathbb{E}\Big[\mathbf{1}_{\{X_{T_D}\geq X_{NF}\}}e^{\sigma W_{T_D}}\Big]-e^{rT_D}C\mathbb{P}\Big(W_{T_D}\geq\frac{\ln\frac{X_{NF}}{X}-(\mu-\frac{\sigma^2}{2})T_D}{\sigma}\Big)\\
&=\frac{Xe^{-(r-\mu)T_D}}{r-\mu}e^{-\frac{\sigma^2T_D}{2}}\mathbb{E}\Big[\mathbf{1}_{\{z\geq-d_2\}}e^{\sigma\sqrt{T_D}z}\Big]-e^{-rT_D}C\mathbb{P}(z\geq-d_2)\\
&=\frac{Xe^{-(r-\mu)T_D}}{r-\mu}\int_{-d_2}^{\infty}\frac{1}{\sqrt{2\pi}}e^{-\frac{(z-\sigma\sqrt{T_D})^2}{2}}\mathrm{d}z-e^{-rT_D}CN(d_2)\\
&=\frac{Xe^{-(r-\mu)T_D}}{r-\mu}N(d_1)-e^{-rT_D}CN(d_2),
\end{aligned}
\tag{71}
$$

where d_1 and d_2 are given by (18) and (19), respectively, and z denotes a standard normal random variable.

The evaluation of the second term on the right-hand side of (16) follows a similar argument:

$$
\begin{aligned}
&\mathbb{E}\Big[\mathbf{1}_{\{X_{T_D}<X_{NF}\}}e^{-rT_D}\Big(\frac{X_{NF}}{r-\mu}-C\Big)\Big(\frac{X_{T_D}}{X_{NF}}\Big)^{\beta}\Big|X_0=X\Big]\\
&=e^{-rT_D}\Big(\frac{X_{NF}}{r-\mu}-C\Big)\Big(\frac{1}{X_{NF}}\Big)^{\beta}\mathbb{E}\Big[\mathbf{1}_{\{X_{T_D}<X_{NF}\}}\big(Xe^{(\mu-\frac{\sigma^2}{2})T_D+\sigma W_{T_D}}\big)^{\beta}\Big]\\
&=e^{-rT_D}\Big(\frac{X_{NF}}{r-\mu}-C\Big)\Big(\frac{Xe^{(\mu-\frac{\sigma^2}{2})T_D}}{X_{NF}}\Big)^{\beta}\mathbb{E}[\mathbf{1}_{\{z<-d_2\}}e^{\beta\sigma\sqrt{T_D}z}]\\
&=e^{-rT_D}\Big(\frac{X_{NF}}{r-\mu}-C\Big)\Big(\frac{Xe^{(\mu-\frac{\sigma^2}{2})T_D}}{X_{NF}}\Big)^{\beta}e^{\frac{\beta^2\sigma^2T_D}{2}}\int_{-\infty}^{-d_2}\frac{1}{\sqrt{2\pi}}e^{-\frac{(z-\beta\sigma\sqrt{T_D})^2}{2}}\mathrm{d}z\\
&=e^{-rT_D}\Big(\frac{X_{NF}}{r-\mu}-C\Big)\Big(\frac{Xe^{(\mu+\frac{(\beta-1)\sigma^2}{2})T_D}}{X_{NF}}\Big)^{\beta}N(d_3)\\
&=\Big(\frac{X_{NF}}{r-\mu}-C\Big)\Big(\frac{X}{X_{NF}}\Big)^{\beta}N(d_3),
\end{aligned}
\tag{72}
$$

where d_3 is given by (20). The last row of (72) results from $\sigma^2\beta(\beta-1)/2+\beta\mu-r=0$. The sum of (71) and (72) amounts to the result in Proposition 5.

A.8 Proof of Proposition 6

First, we evaluate the first term on the right-hand side of (22):

$$
\mathbb{E}\Big[\mathbf{1}_{\{X_{\tau_D}\geq X_{NU}\}}e^{-r\tau_D}\Big(\frac{X_{\tau_D}}{r-\mu}-C\Big)\Big|X_0=X\Big].
\tag{73}
$$

This portion of the option to invest becomes worthless if the demand at the deregulation time falls short of the investment threshold. Thus, its value for $X<X_{NU}$, denoted by $\underline{F}(X)$, should satisfy

$$
r\underline{F}(X)=\mathcal{L}\underline{F}(X)+\lambda_D\{-\underline{F}(X)\},
\tag{74}
$$

subject to

$$
\underline{F}(0)=0,
\tag{75}
$$

and other boundary conditions illustrated below. A general solution to (74) with the boundary condition (75) is

$$
\underline{F}(X)=A_1X^{\beta_D},
\tag{76}
$$

where β_D is given by (24).

The firm's option to invest described in (73) is exercised immediately if the demand at the deregulation timing exceeds the investment threshold X_{NU}. Thus, its value for $X \geq X_{NU}$, denoted by $\bar{F}(X)$, should satisfy

$$r\bar{F}(X) = \mathcal{L}\bar{F}(X) + \lambda_D\Big\{\frac{X}{r-\mu} - C - \bar{F}(X)\Big\}, \tag{77}$$

subject to

$$\lim_{X\to\infty} \bar{F}(X) = \frac{\lambda_D X}{(r-\mu)(r+\lambda_D-\mu)} - \frac{\lambda_D C}{r+\lambda_D}, \tag{78}$$

and the other boundary conditions described below. A general solution to (77) with the boundary condition (78) is

$$\bar{F}(X) = \frac{\lambda_D X}{(r-\mu)(r+\lambda_D-\mu)} - \frac{\lambda_D C}{r+\lambda_D} + A_2 X^{\gamma_D}, \tag{79}$$

where γ_D is given by (25).

Meanwhile, (76) and (79) are subject to the following boundary conditions:

$$\underline{F}(X_{NU}) = \bar{F}(X_{NU}), \tag{80}$$

$$\frac{\partial \underline{F}}{\partial X}\Big|_{X=X_{NU}} = \frac{\partial \bar{F}}{\partial X}\Big|_{X=X_{NU}}. \tag{81}$$

Substituting (76) and (79) into (80) and (81), we can derive A_1 and A_2, which amounts to the following results:

$$\underline{F}(X) = \frac{1}{\beta_D-\gamma_D}\Big[\frac{(1-\gamma_D)\lambda_D X_{NU}}{(r-\mu)(r+\lambda_D-\mu)} + \frac{\gamma_D\lambda_D C}{r+\lambda_D}\Big]\Big(\frac{X}{X_{NU}}\Big)^{\beta_D}, \tag{82}$$

$$\bar{F}(X) = \frac{\lambda_D X}{(r-\mu)(r+\lambda_D-\mu)} - \frac{\lambda_D C}{r+\lambda_D} - \frac{1}{\beta_D-\gamma_D}\Big[\frac{(\beta_D-1)\lambda_D X_{NU}}{(r-\mu)(r+\lambda_D-\mu)} - \frac{\beta_D\lambda_D C}{r+\lambda_D}\Big]\Big(\frac{X}{X_{NU}}\Big)^{\gamma_D}. \tag{83}$$

Now let us proceed with the evaluation of the second term on the right-hand side of (22):

$$\mathbb{E}\Big[\mathbf{1}_{\{X_{\tau_D}<X_{NU}\}} e^{-r\tau_D}\Big(\frac{X_{NU}}{r-\mu} - C\Big)\Big(\frac{X_{\tau_D}}{X_{NU}}\Big)^{\beta}\Big|X_0 = X\Big]. \tag{84}$$

This portion of the option becomes an American option described in (21) if the demand at the deregulation timing is below the investment threshold X_{NU}; it becomes worthless otherwise. Thus, its value for $X < X_{NU}$ and $X \geq X_{NU}$, denoted by $\underline{G}(X)$ and $\bar{G}(X)$, respectively, should satisfy

$$r\underline{G}(X) = \mathcal{L}\underline{G}(X) + \lambda_D\Big\{\Big(\frac{X_{NU}}{r-\mu} - C\Big)\Big(\frac{X}{X_{NU}}\Big)^{\beta} - \underline{G}(X)\Big\}, \tag{85}$$

$$r\bar{G}(X) = \mathcal{L}\bar{G}(X) + \lambda_D\{-\bar{G}(X)\}, \tag{86}$$

subject to

$$\underline{G}(0) = 0, \tag{87}$$

$$\lim_{X\to\infty} \bar{G}(X) = 0, \tag{88}$$

$$\underline{G}(X_{NU}) = \bar{G}(X_{NU}), \tag{89}$$

$$\frac{\partial \underline{G}}{\partial X}\Big|_{X=X_{NU}} = \frac{\partial \bar{G}}{\partial X}\Big|_{X=X_{NU}}. \tag{90}$$

Following the same arguments from the derivation of (82) and (83), we can obtain

$$\underline{G}(X) = \Big(\frac{X_{NU}}{r-\mu} - C\Big)\Big\{\Big(\frac{X}{X_{NU}}\Big)^{\beta} - \frac{\beta-\gamma_D}{\beta_D-\gamma_D}\Big(\frac{X}{X_{NU}}\Big)^{\beta_D}\Big\}, \tag{91}$$

$$\bar{G}(X) = \frac{\beta_D-\beta}{\beta_D-\gamma_D}\Big(\frac{X_{NU}}{r-\mu} - C\Big)\Big(\frac{X}{X_{NU}}\Big)^{\gamma_D}. \tag{92}$$

The sum of (82) and (91) and that of (83) and (92) correspond to the upper and lower cases of (23), respectively.

A.9 Proof of Proposition 7

It is well known from the Black-Scholes formula that d_2 given in (19) is directly associated with the probability of a European call option being in-the-money at the maturity. Specifically, the proof of Proposition 5 in Appendix A.7 shows that the probability of investment triggered at the fixed deregulation timing is as follows:

$$\hat{\Gamma}_F = \mathbb{P}\big(X_{T_D} \geq X_{NF} \big| X_0 = X\big) = \mathbb{P}(z \geq -d_2) = N(d_2). \tag{93}$$

Using the cumulative distribution function of the maximum process of geometric Brownian motion (e.g., Privault (2023)), the probability of not reaching the investment threshold until the fixed deregulation timing is

$$\begin{aligned}
&\mathbb{P}(\max_{t\in[0,T_D]} X_t < X_{NF} | X_0 = X) \\
&= N\Big(-\frac{\ln\frac{X}{X_{NF}} + (\mu - \frac{\sigma^2}{2})T_D}{\sigma\sqrt{T_D}}\Big) - \Big(\frac{X}{X_{NF}}\Big)^{1-\frac{2\mu}{\sigma^2}} N\Big(\frac{\ln\frac{X}{X_{NF}} - (\mu - \frac{\sigma^2}{2})T_D}{\sigma\sqrt{T_D}}\Big) \\
&= 1 - N(d_2) - \Big(\frac{X}{X_{NF}}\Big)^{1-\frac{2\mu}{\sigma^2}} N(\bar{d}_2),
\end{aligned} \tag{94}$$

where $\bar{d}_2$ is given by (30). Thus, the probability of investment not triggered by deregulation yet delayed by fixed-duration regulation is

$$\begin{aligned}
\bar{\Gamma}_F &= \mathbb{P}(X_{T_D} < X_{NF} | X_0 = X) - \mathbb{P}(\max_{t\in[0,T_D]} X_t < X_{NF} | X_0 = X) \\
&= 1 - N(d_2) - \Big\{1 - N(d_2) - \Big(\frac{X}{X_{NF}}\Big)^{1-\frac{2\mu}{\sigma^2}} N(\bar{d}_2)\Big\} \\
&= \Big(\frac{X}{X_{NF}}\Big)^{1-\frac{2\mu}{\sigma^2}} N(\bar{d}_2).
\end{aligned} \tag{95}$$

Meanwhile, the probability density function of geometric Brownian motion stopped at exponential timing is well-known (e.g., Borodin and Salminen (2015)). Thus, the probability of investment triggered at uncertain deregulation timing can be evaluated as follows:

$$\hat{\Gamma}_U = \mathbb{P}\big(X_{\tau_D} \geq X_{NU} \big| X_0 = X\big) = \int_{X_{NU}}^{\infty} \frac{-\hat{\beta}_D\hat{\gamma}_D}{z(\hat{\beta}_D - \hat{\gamma}_D)}\Big(\frac{X}{z}\Big)^{\hat{\beta}_D} \mathrm{d}z = \frac{-\hat{\gamma}_D}{\hat{\beta}_D - \hat{\gamma}_D}\Big(\frac{X}{X_{NU}}\Big)^{\hat{\beta}_D}, \tag{96}$$

where $\hat{\beta}_D$ and $\hat{\gamma}_D$ are given by (31) and (32), respectively. Using the cumulative distribution function of the maximum of geometric Bronwnian motion stopped at exponential timing (e.g., Borodin and Salminen (2015)), the probability of not hitting the investment threshold before the uncertain deregulation timing is

$$\mathbb{P}(\max_{t\in[0,\tau_D]} X_t < X_{NU} | X_0 = X) = 1 - \Big(\frac{X}{X_{NU}}\Big)^{\hat{\beta}_D}. \tag{97}$$

Thus, the probability of investment not triggered by deregulation yet delayed by uncertain regulation is

$$\begin{aligned}
\bar{\Gamma}_U &= \mathbb{P}(X_{\tau_D} < X_{NU} | X_0 = X) - \mathbb{P}(\max_{t\in[0,\tau_D]} X_t < X_{NU} | X_0 = X) \\
&= 1 + \frac{\hat{\gamma}_D}{\hat{\beta}_D - \hat{\gamma}_D}\Big(\frac{X}{X_{NU}}\Big)^{\hat{\beta}_D} - \Big\{1 - \Big(\frac{X}{X_{NU}}\Big)^{\hat{\beta}_D}\Big\} \\
&= \Big(1 + \frac{\hat{\gamma}_D}{\hat{\beta}_D - \hat{\gamma}_D}\Big)\Big(\frac{X}{X_{NU}}\Big)^{\hat{\beta}_D}.
\end{aligned} \tag{98}$$

A.10 Proof of Corollary 2

A straightforward calculation yields

$$\begin{aligned}
\frac{\partial \Gamma_F}{\partial T_D} &= \frac{\partial N(d_2)}{\partial d_2}\frac{\partial d_2}{\partial T_D} + \Big(\frac{X}{X_{NF}}\Big)^{1-\frac{2\mu}{\sigma^2}} \frac{\partial N(\bar{d}_2)}{\partial \bar{d}_2}\frac{\partial \bar{d}_2}{\partial T_D} \\
&= \frac{1}{\sqrt{2\pi}} e^{-\frac{(d_2)^2}{2}}\Big(-\frac{\bar{d}_2}{2T_D}\Big) + \Big(\frac{X}{X_{NF}}\Big)^{1-\frac{2\mu}{\sigma^2}} \frac{1}{\sqrt{2\pi}} e^{-\frac{(\bar{d}_2)^2}{2}}\Big(-\frac{d_2}{2T_D}\Big).
\end{aligned} \tag{99}$$

Thus, $\partial\Gamma_F/\partial T_D > 0$ holds if both d_2 and $\bar{d}_2$ are negative, which is equivalent to (33).

Meanwhile, some tedious algebra yields

$$\frac{\partial\Gamma_U}{\partial\lambda_D} = \frac{\partial\Gamma_U}{\partial\hat{\beta}_D}\frac{\partial\hat{\beta}_D}{\partial\lambda_D} = \ln\frac{X}{X_{NU}}\Big(\frac{X}{X_{NU}}\Big)^{\hat{\beta}_D}\frac{1}{\sigma^2\sqrt{(\frac{1}{2}-\frac{\mu}{\sigma^2})^2+\frac{2\lambda_D}{\sigma^2}}} < 0. \tag{100}$$

A.11 Proof of Corollary 3

Following the same arguments as in Proposition 5, we have

$$\hat{\phi}_{NF}(X) = \mathbb{E}\big[\mathbf{1}_{\{X_{T_D}\geq X_{NF}\}}e^{-rT_D}\big|X_0 = X\big] = e^{-rT_D}\mathbb{P}\big(Xe^{(\mu-\frac{\sigma^2}{2})T_D+\sigma W_{T_D}} \geq X_{NF}\big) = e^{-rT_D}N(d_2), \tag{101}$$

and

$$\bar{\phi}_{NF}(X) = \mathbb{E}\big[\mathbf{1}_{\{X_{T_D}<X_{NF}\}}e^{-rT_{NF}}\big|X_0 = X\big] = e^{-rT_D}\Big(\frac{Xe^{(\mu-\frac{\sigma^2}{2})T_D}}{X_{NF}}\Big)^{\beta}\mathbb{E}\big[\mathbf{1}_{\{X_{T_D}<X_{NF}\}}e^{\beta\sigma W_{T_D}}\big] = \Big(\frac{X}{X_{NF}}\Big)^{\beta}N(d_3), \tag{102}$$

which proves (34) with (35) and (36).

Likewise, the derivation of (37) with (38) and (39) follows similar arguments as in Proposition 6, and let us derive (38). The state price for $X < X_{NU}$, denoted by $\underline{F}(X)$, should satisfy (74) subject to (75) and the other boundary conditions described below. Thus, its general solution in this region is (76). The state price for $X \geq X_{NU}$, denoted by $\bar{F}(X)$, should satisfy

$$r\bar{F}(X) = \mathcal{L}\bar{F}(X) + \lambda_D\{1-\bar{F}(X)\}, \tag{103}$$

subject to

$$\lim_{X\to\infty}\bar{F}(X) = \frac{\lambda_D}{r+\lambda_D}, \tag{104}$$

and the other boundary conditions below. A general solution to (103) with the boundary condition (104) is

$$\bar{F}(X) = \frac{\lambda_D}{r+\lambda_D} + A_2X^{\gamma_D}. \tag{105}$$

The value-matching and smooth-pasting conditions of $\underline{F}(X)$ and $\bar{F}(X)$ at X_{NU} yield (38).

The derivation of (39) follows similar arguments, and we omit it for brevity.

A.12 Proof of Lemma 2

Suppose that $T_B < T_D$. If the firm invests at $T \in [0, T_D - T_B)$, the project is finished at $T + T_B(< T_D)$ but the firm cannot raise revenue until T_D. Thus, by Lemma 1, the firm never invests before $T_D - T_B$. If the firm invests at $T \in [T_D - T_B, \infty)$, by the time the project is finished at $T + T_B(\geq T_D)$, the regulation is lifted with certainty. Thus, it can be optimal for the firm to invest after $T_D - T_B$, even if it is under regulation.

Now, suppose that $T_B \geq T_D$. Even if the firm invests at the initial timing (i.e., $t = 0$), the regulation is lifted by the time the project is finished. That is, whether to invest immediately or not is not relevant to the timing of deregulation.

Thus, we can conclude that the firm decides whether to invest immediately or not at $\bar{T}_D := \max(T_D - T_B, 0)$.

A.13 Proof of Proposition 8

The proof of this proposition is very similar to that of Proposition 5 in Appendix A.7. The first term on the right-hand side of (41) is

$$\begin{aligned}
&\mathbb{E}\Big[\mathbf{1}_{\{X_{\bar{T}_D}\geq X_{FF}\}}e^{-r\bar{T}_D}\Big(\frac{X_{\bar{T}_D}e^{-(r-\mu)T_B}}{r-\mu} - C\Big)\Big|X_0 = X\Big] \\
&= \frac{Xe^{-(r-\mu)(\bar{T}_D+T_B)}}{r-\mu}e^{-\frac{\sigma^2\bar{T}_D}{2}}\mathbb{E}\Big[\mathbf{1}_{\{X_{\bar{T}_D}\geq X_{FF}\}}e^{\sigma W_{\bar{T}_D}}\Big] - e^{-r\bar{T}_D}C\mathbb{P}\Big(W_{\bar{T}_D} \geq \frac{\ln\frac{X_{FF}}{X} - (\mu-\frac{\sigma^2}{2})\bar{T}_D}{\sigma}\Big) \\
&= \frac{Xe^{-(r-\mu)(\bar{T}_D+T_B)}}{r-\mu}e^{-\frac{\sigma^2\bar{T}_D}{2}}\mathbb{E}\Big[\mathbf{1}_{\{z\geq -d_5\}}e^{\sigma\sqrt{\bar{T}_D}z}\Big] - e^{-r\bar{T}_D}C\mathbb{P}(z \geq -d_5) \\
&= \frac{Xe^{-(r-\mu)(\bar{T}_D+T_B)}}{r-\mu}N(d_4) - e^{-r\bar{T}_D}CN(d_5),
\end{aligned} \tag{106}$$

where d_4 and d_5 are given by (43) and (44), respectively, and z denotes a standard normal random variable.

Similarly, the second term on the right-hand side of (41) is

$$
\begin{aligned}
&\mathbb{E}\Big[\mathbf{1}_{\{X_{\bar{T}_D}<X_{FF}\}}e^{-r\bar{T}_D}\Big(\frac{X_{FF}e^{-(r-\mu)T_B}}{r-\mu}-C\Big)\Big(\frac{X_{\bar{T}_D}}{X_{FF}}\Big)^{\beta}\Big|X_0=X\Big] \\
&=e^{-r\bar{T}_D}\Big(\frac{X_{FF}e^{-(r-\mu)T_B}}{r-\mu}-C\Big)\Big(\frac{Xe^{(\mu-\frac{\sigma^2}{2})\bar{T}_D}}{X_{FF}}\Big)^{\beta}\mathbb{E}[\mathbf{1}_{\{X_{\bar{T}_D}<X_{FF}\}}e^{\beta\sigma W_{\bar{T}_D}}] \\
&=e^{-r\bar{T}_D}\Big(\frac{X_{FF}e^{-(r-\mu)T_B}}{r-\mu}-C\Big)\Big(\frac{Xe^{(\mu-\frac{\sigma^2}{2})\bar{T}_D}}{X_{FF}}\Big)^{\beta}\mathbb{E}[\mathbf{1}_{\{z<-d_5\}}e^{\beta\sigma\sqrt{\bar{T}_D}z}] \\
&=e^{-r\bar{T}_D}\Big(\frac{X_{FF}e^{-(r-\mu)T_B}}{r-\mu}-C\Big)\Big(\frac{Xe^{(\mu+\frac{(\beta-1)\sigma^2}{2})\bar{T}_D}}{X_{FF}}\Big)^{\beta}N(d_6) \\
&=\Big(\frac{X_{FF}e^{-(r-\mu)T_B}}{r-\mu}-C\Big)\Big(\frac{X}{X_{FF}}\Big)^{\beta}N(d_6), \qquad (107)
\end{aligned}
$$

where d_6 is given by (45).

The sum of (106) and (107) amounts to the result in Proposition 8.

A.14 Proof of Proposition 9

At the investment timing under regulation, the firm's expected profits are

$$
V(X)=\mathbb{E}\Big[\mathbf{1}_{\{\tau_D\le\tau_B\}}\int_{\tau_B}^{\infty}e^{-rt}X_t\mathrm{d}t+\mathbf{1}_{\{\tau_B<\tau_D\}}\int_{\tau_D}^{\infty}e^{-rt}X_t\mathrm{d}t-C\Big|X_0=X\Big]. \qquad (108)
$$

The first term in (108) is

$$
\begin{aligned}
&\int_0^{\infty}\Big\{\int_{\tau_D}^{\infty}\Big(\int_{\tau_B}^{\infty}e^{-rt}X_t\mathrm{d}t\Big)\lambda_Be^{-\lambda_B\tau_B}\mathrm{d}\tau_B\Big\}\lambda_De^{-\lambda_D\tau_D}\mathrm{d}\tau_D \qquad (109)\\
=&\int_0^{\infty}\Big\{\int_{\tau_D}^{\infty}\frac{\lambda_BX}{r-\mu}e^{-(r+\lambda_B-\mu)\tau_B}\mathrm{d}\tau_B\Big\}\lambda_De^{-\lambda_D\tau_D}\mathrm{d}\tau_D \\
=&\frac{\lambda_B\lambda_DX}{(r-\mu)(r+\lambda_B-\mu)}\int_0^{\infty}e^{-(r+\lambda_B+\lambda_D-\mu)\tau_D}\mathrm{d}\tau_D \\
=&\frac{\lambda_B\lambda_DX}{(r-\mu)(r+\lambda_B-\mu)(r+\lambda_B+\lambda_D-\mu)}. \qquad (110)
\end{aligned}
$$

Similarly, the second term in (108) can be calculated as

$$
\frac{\lambda_B\lambda_DX}{(r-\mu)(r+\lambda_D-\mu)(r+\lambda_B+\lambda_D-\mu)}, \qquad (111)
$$

and thus,

$$
V(X)=\frac{\lambda_B\lambda_DX}{(r-\mu)(r+\lambda_B+\lambda_D-\mu)}\Big(\frac{1}{r+\lambda_B-\mu}+\frac{1}{r+\lambda_D-\mu}\Big)-C. \qquad (112)
$$

From Lemma 1, we can conjecture that $X_{UU}^D\le X_{UU}^R$ holds regarding the two investment thresholds. Suppose the regulation has not been lifted yet. For $X<X_{UU}^D$, the firm value switches to $V_{UU}^D(X)$ in (46), which is equivalent to $V_{UN}(X)$ in (7), as soon as the regulation is lifted. Thus, the firm value in this region, denoted by $\underline{F}(X)$, should satisfy

$$
r\underline{F}(X)=\mathcal{L}\underline{F}(X)+\lambda_D\Big\{\Big[\frac{\lambda_BX_{UU}^D}{(r-\mu)(r+\lambda_B-\mu)}-C\Big]\Big(\frac{X}{X_{UU}^D}\Big)^{\beta}-\underline{F}(X)\Big\}, \qquad (113)
$$

subject to

$$
\underline{F}(0)=0. \qquad (114)
$$

Thus, a general solution to (113) is

$$
\underline{F}(X)=\Big[\frac{\lambda_BX_{UU}^D}{(r-\mu)(r+\lambda_B-\mu)}-C\Big]\Big(\frac{X}{X_{UU}^D}\Big)^{\beta}+A_1X^{\beta_D}. \qquad (115)
$$

For $X^D_{UU} \le X < X^R_{UU}$, the firm invests immediately if the regulation is lifted. Thus, the firm value in this region, denoted by $\bar{F}(X)$, should satisfy

$$r\bar{F}(X) = \mathcal{L}\bar{F}(X) + \lambda_D\Big\{\frac{\lambda_B X}{(r-\mu)(r+\lambda_B-\mu)} - C - \bar{F}(X)\Big\}, \tag{116}$$

subject to some boundary conditions, which we describe shortly. A general solution to (116) is

$$\bar{F}(X) = \frac{\lambda_B\lambda_D X}{(r-\mu)(r+\lambda_B-\mu)(r+\lambda_D-\mu)} - \frac{\lambda_D C}{r+\lambda_D} + A_2X^{\beta_D} + A_3X^{\gamma_D}. \tag{117}$$

Meanwhile, (113) and (116) are subject to the following boundary conditions:

$$\underline{F}(X^D_{UU}) = \bar{F}(X^D_{UU}), \tag{118}$$

$$\frac{\partial \underline{F}}{\partial X}\Big|_{X=X^D_{UU}} = \frac{\partial \bar{F}}{\partial X}\Big|_{X=X^D_{UU}}, \tag{119}$$

$$\bar{F}(X^R_{UU}) = V(X^R_{UU}), \tag{120}$$

$$\frac{\partial \bar{F}}{\partial X}\Big|_{X=X^R_{UU}} = \frac{\partial V}{\partial X}\Big|_{X=X^R_{UU}}. \tag{121}$$

Substituting (112), (115), and (117) into (118) through (121), we can derive A_1, A_2, A_3, and X^R_{UU}, which amounts to the results in Proposition 9.

A.15 Proof of Corollary 4

Following the same arguments as in Proposition 8, we have

$$\hat{\phi}_{FF}(X) = \mathbb{E}\big[\mathbf{1}_{\{X_{\bar{T}_D}\ge X_{FF}\}}e^{-r\bar{T}_D}\big|X_0 = X\big] = e^{-r\bar{T}_D}\mathbb{P}\big(Xe^{(\mu-\frac{\sigma^2}{2})\bar{T}_D+\sigma W_{\bar{T}_D}} \ge X_{FF}\big) = e^{-r\bar{T}_D}N(d_5), \tag{122}$$

and

$$\bar{\phi}_{FF}(X) = \mathbb{E}\big[\mathbf{1}_{\{X_{\bar{T}_D}<X_{FF}\}}e^{-rT_{FF}}\big|X_0 = X\big] = e^{-r\bar{T}_D}\Big(\frac{Xe^{(\mu-\frac{\sigma^2}{2})\bar{T}_D}}{X_{FF}}\Big)^{\beta}\mathbb{E}\big[\mathbf{1}_{\{X_{\bar{T}_D}<X_{FF}\}}e^{\beta\sigma W_{\bar{T}_D}}\big] = \Big(\frac{X}{X_{FF}}\Big)^{\beta}N(d_6), \tag{123}$$

which proves (50) with (51) and (52).

Likewise, the derivation of (53) with (54) through (56) follows similar arguments as in Proposition 9, and let us calculate (54). The state price for $t < \tau_D$ and $X < X^D_{UU}$, denoted by $\underline{F}(X)$, should satisfy (74) subject to (75) and the other boundary conditions described below. A general solution to (74) with the boundary condition (75) is (76). The state price for $t < \tau_D$ and $X^D_{UU} \le X < X^R_{UU}$, denoted by $\bar{F}(X)$, should satisfy (103) subject to boundary conditions introduced below. Its general solution is

$$\bar{F}(X) = \frac{\lambda_D}{r+\lambda_D} + A_2X^{\beta_D} + A_3X^{\gamma_D}, \tag{124}$$

and $\underline{F}(X)$ and $\bar{F}(X)$ are subject to value-matching and smooth-pasting conditions at X^D_{UU} and $\bar{F}(X^R_{UU}) = 0$. A straightforward calculation yields (54).

The derivation of (55) and (56) follows similar arguments, and we omit them for brevity.

A.16 Proof of Proposition 10

Suppose $T_D \le T_B$ such that $\bar{T}_D = 0$. As discussed for Proposition 8, $\lim_{\bar{T}_D\to 0} d_5 = -\infty$ and $\lim_{\bar{T}_D\to 0} d_6 = \infty$ hold, unless the initial demand is sufficiently high such that investment is triggered at the initial timing (i.e., $X > X_{FF}$). Thus, $\hat{\phi}_{FF}(X) = 0$ and $\bar{\phi}_{FF}(X) = (X/X_{FF})^{\beta}$ hold for $\bar{T}_D = 0$. Note that $X_{FF} = X_{FN}$ and $\phi_{FN}(X) = (X/X_{FN})^{\beta}$ amount to $\tilde{\phi}_{FF}(X) = 0$, from which $V_{FF}(X) = V_{FN}(X)$ is obvious.

References

Abel, A. (1983), 'Optimal investment under uncertainty', *American Economic Review* **73**(1), 228–233.

Abel, A. and Eberly, J. (1999), 'The effects of irreversibility and uncertainty on capital accumulation', *Journal of Monetary Economics* **44**(3), 339–377.

Alesina, A., Nicoletti, G., Ardagna, S. and Schiantarelli, F. (2005), 'Regulation and investment', *Journal of the European Economic Association* **3**(4), 791–825.

Alvarez, L. and Keppo, J. (2002), 'The impact of delivery lags on irreversible investment under uncertainty', *European Journal of Operational Research* **136**(1), 173–180.

Bar-Ilan, A. and Strange, W. (1996*a*), 'Investment lags', *American Economic Review* **86**(3), 610–622.

Bar-Ilan, A. and Strange, W. (1996*b*), 'Urban development with lags', *Journal of Urban Economics* **39**(1), 87–113.

Bar-Ilan, A. and Strange, W. (1998), 'A model of sequential investment', *Journal of Economic Dynamics and Control* **22**(3), 437–463.

Bar-Ilan, A., Sulem, A. and Zanello, A. (2002), 'Time-to-build and capacity choice', *Journal of Economic Dynamics and Control* **26**(1), 69–98.

Barone, G. and Cingano, F. (2011), 'Service regulation and growth: Evidence from OECD countries', *Economic Journal* **121**(555), 931–957.

Bo, H. and Lensin, R. (2005), 'Is the investment-uncertainty relationship nonlinear? An empirical analysis for the Netherlands', *Economica* **72**, 307–331.

Borodin, A. and Salminen, P. (2015), *Handbook of Brownian Motions — Facts and Formulae*, 2nd edn, Birkhäuser.

Boston, W. and Kostov, N. (2021), 'Global chip crisis hits auto makers hard', *The Wall Street Journal* . October 28, 2021.

Branstetter, L., Lima, F., Taylor, L. and Venâncio, A. (2014), 'Do entry regulations deter entrepreneurship and job creation? evidence from recent reforms in Portugal', *Economic Journal* **124**(577), 805–832.

Bushey, C. and Pfeifer, S. (2023), 'US air taxi chief says european regulation 'not good for business'', *Financial Times* . June 11, 2023.

Caballero, R. (1991), 'On the sign of the investment-uncertainty relationship', *American Economic Review* **81**(1), 279–288.

Carruth, A., Dickerson, A. and Henley, A. (2000), 'What do we know about investment under uncertainty?', *Journal of Economic Surveys* **14**(2), 119–153.

Del Boca, A., Himmelberg, C., Galeotti, M. and Rota, P. (2008), 'Investment and time to plan and build: A comparison of structures vs. equipment in a panel of Italian firms', *Journal of the European Economic Association* **6**(4), 864–889.

Djankov, S., La Porta, R., Lopez-de-Silanes, F. and Shleifer, A. (2002), 'The regulation of entry', *Quarterly Journal of Economics* **117**(1), 1–37.

Engau, C. and Hoffmann, V. (2009), 'Effects of regulatory uncertainty on corporate strategy—an analysis of firms' responses to uncertainty about post-Kyoto policy', *Environmental Science and Policy* **12**(7), 766–777.

Fabrizio, K. (2013), 'The effect of regulatory uncertainty on investment: Evidence from renewable energy generation', *Journal of Law, Economics, and Organization* **29**(4), 765–798.

Ferderer, J. (1993), 'The impact of uncertainty on aggregate investment spending: An empirical analysis', *Journal of Money, Credit and Banking* **25**(1), 30–48.

Financial Times Lex (2023), 'Robotaxis: piecemeal regulation strews speed bumps to US rollout', *Financial Times* . August 11, 2023.

Guiso, L. and Parigi, G. (1999), 'Investment and demand uncertainty', *Quarterly Journal of Economics* **114**(1), 185–227.

Gulen, H. and Ion, M. (2016), 'Policy uncertainty and corporate investment', *Review of Financial Studies* **29**(3), 523–564.

Hansen, E. and Wagner, R. (2017), 'Stockpiling cash when it takes time to build: exploring price differentials in a commodity boom', *Journal of Banking and Finance* **77**, 197–212.

Hartman, R. (1972), 'The effects of price and cost uncertainty on investment', *Journal of Economic Theory* **5**(2), 258–266.

Hartman, R. (1973), 'Adjustment costs, price and wage uncertainty, and investment', *Review of Economic Studies* **40**(2), 259–267.

Hoffmann, V., Trautmann, T. and Hamprecht, J. (2009), 'Regulatory uncertainty: A reason to postpone investments? Not necessarily', *Journal of Management Studies* **46**(7), 1227–1253.

Hollinger, P. and Waters, R. (2022), 'Chip makers face two-year shortage of critical equipment', *Financial Times* . March 21, 2022.

Huizinga, J. (1993), 'Inflation uncertainty, relative price uncertainty, and investment in U.S. manufacturing', *Journal of Money, Credit and Banking* **25**(3), 521–549.

Jeon, H. (2021*a*), 'Investment and financing decisions in the presence of time-to-build', *European Journal of Operational Research* **288**(3), 1068–1084.

Jeon, H. (2021*b*), 'Investment timing and capacity decisions with time-to-build in a duopoly market', *Journal of Economic Dynamics and Control* **122**, 104028.

Jeon, H. (2023), 'Time-to-build and capacity expansion', *Annals of Operations Research* **328**, 1461–1494.

Kalouptsidi, M. (2014), 'Time to build and fluctuations in bulk shipping', *American Economic Review* **104**(2), 564–608.

Klapper, L., Laeven, L. and Rajan, R. (2006), 'Entry regulation as a barrier to entrepreneurship', *Journal of Financial Economics* **82**(3), 591–629.

Koeva, P. (2000), The facts about time-to-build. IMF working paper.

Leahy, J. and Whited, T. (1996), 'The effect of uncertainty on investment: Some stylized facts', *Journal of Money, Credit and Banking* **28**(1), 64–83.

Lopez, J., Sakhel, A. and Busch, T. (2017), 'Corporate investments and environmental regulation: The role of regulatory uncertainty, regulation-induced uncertainty, and investment history', *European Management Journal* **35**(1), 91–101.

Majd, S. and Pindyck, R. (1987), 'Time to build, option value, and investment decisions', *Journal of Financial Economics* **18**(1), 7–27.

Marcus, A. (1981), 'Policy uncertainty and technological innovation', *Academy of Management Review* **6**(3), 443–448.

Marmer, V. and Slade, M. (2018), 'Investment and uncertainty with time to build: Evidence from entry into U.S. copper mining', *Journal of Economic Dynamics and Control* **95**, 233–254.

McDonald, R. and Siegel, D. (1986), 'The value of waiting to invest', *Quarterly Journal of Economics* **101**(4), 707–727.

Meinen, P. and Roehe, O. (2017), 'On measuring uncertainty and its impact on investment: Cross-country evidence from the euro area', *European Economic Review* **92**, 161–179.

Nakamura, T. (1999), 'Risk-aversion and the uncertainty-investment relationship: A note', *Journal of Economic Behavior & Organization* **38**(3), 357–363.

Oh, H. and Yoon, C. (2020), 'Time to build and the real-options channel of residential investment', *Journal of Financial Economics* **135**(1), 255–269.

Pacheco-de-Almeida, G. and Zemsky, P. (2003), 'The effect of time-to-build on strategic investment under uncertainty', *RAND Journal of Economics* **34**(1), 166–182.

Pindyck, R. (1988), 'Irreversible investment, capacity choice, and the value of the firm', *American Economic Review* **78**(5), 969–985.

Pindyck, R. (1993), 'A note on competitive investment under uncertainty', *American Economic Review* **83**(1), 273–277.

Privault, N. (2023), *Introduction to Stochastic Finance with Market Examples*, 2nd edn, CRC Press.

Rothschild, M. and Stiglitz, J. (1970), 'Increasing risk: I. A definition', *Journal of Economic Theory* **2**(3), 225–243.

Salomon, R. and Martin, X. (2008), 'Learning, knowledge transfer, and technology implementation performance: A study of time-to-build in the global semiconductor industry', *Management Science* **54**(7), 1266–1280.

Saltari, E. and Ticchi, D. (2007), 'Risk aversion, intertemporal substitution, and the aggregate investment-uncertainty relationship', *Journal of Monetary Economics* **54**(3), 622–648.

Sarkar, S. (2000), 'On the investment-uncertainty relationship in a real options model', *Journal of Economic Dynamics and Control* **24**(2), 219–225.

Tsoukalas, J. (2011), 'Time to build capital: Revisiting investment-cash-flow sensitivities', *Journal of Economic Dynamics and Control* **35**(7), 1000–1016.

Weeds, H. (2002), 'Strategic delay in a real options model of R&D competition', *Review of Economic Studies* **69**(3), 729–747.

Zhou, C. (2000), 'Time-to-build and investment', *Review of Economics and Statistics* **82**(2), 273–282.